GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

# Scalar - Induced Gravitational Waves

& probing beyond GR theories

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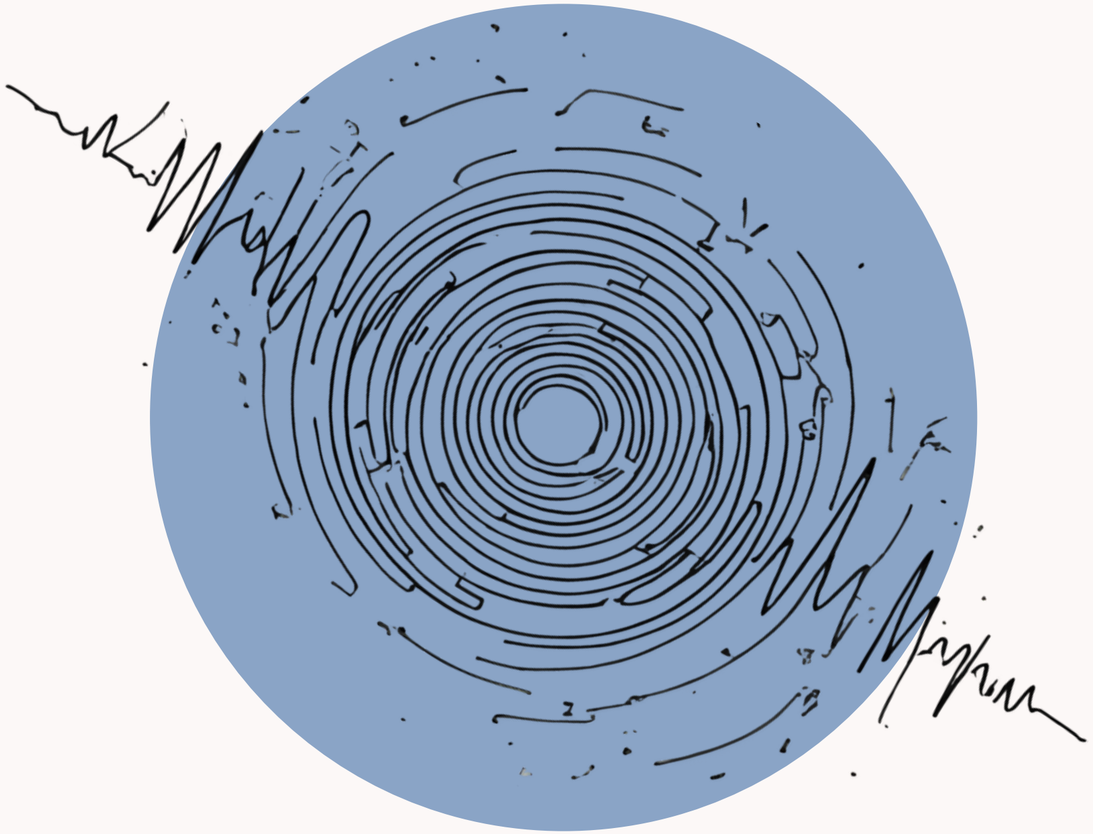

arXiv:2502.20137

AK, Marisol Traforetti, Andrea Maselli, Sabino Matarrese & Angelo Ricciardione

Korea–Italy Workshop on Multi-Messenger Astrophysics and Cosmology – 10.2026

# Probes of Beyond - GR theories

Cosmic Microwave Background



Large Scale Structures



Gravitational Waves produced from BBH and BNS mergers

References: arXiv:2212.09094 & arXiv:2004.01632

# Probes of Beyond - GR theories

Cosmic Microwave Background

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Large Scale Structures

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Gravitational Waves produced from BBH and BNS mergers

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Stochastic Gravitational Wave Background

References: arXiv:2212.09094 & arXiv:2004.01632

# Stochastic GW Background

Inflationary cosmology predicts the generation of quantum vacuum fluctuations

Scalar perturbations

Observed in CMB temperature anisotropies and density fluctuations

Vector perturbations

Correspond to vorticity modes that decay; no detectable imprint in observations

Tensor perturbations

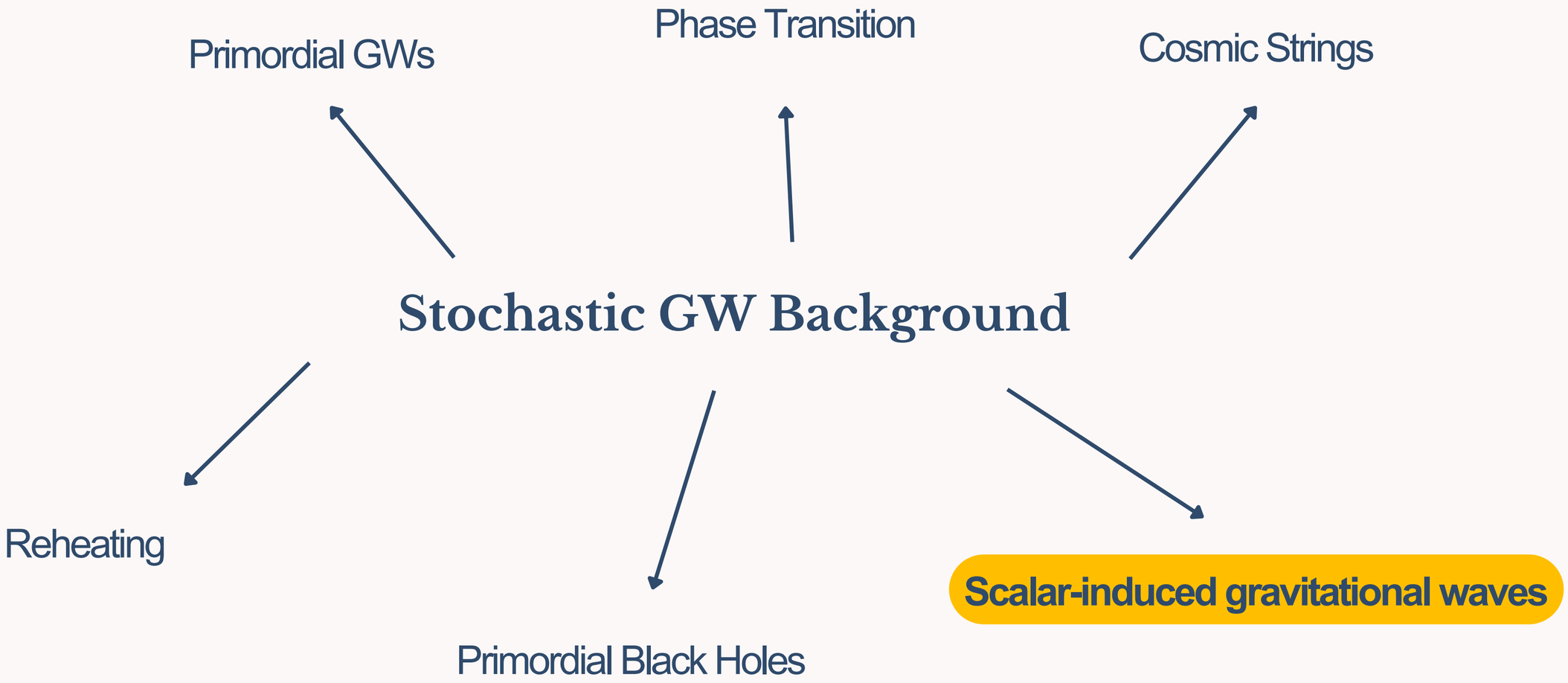
Observed as gravitational waves

# Stochastic GW Background

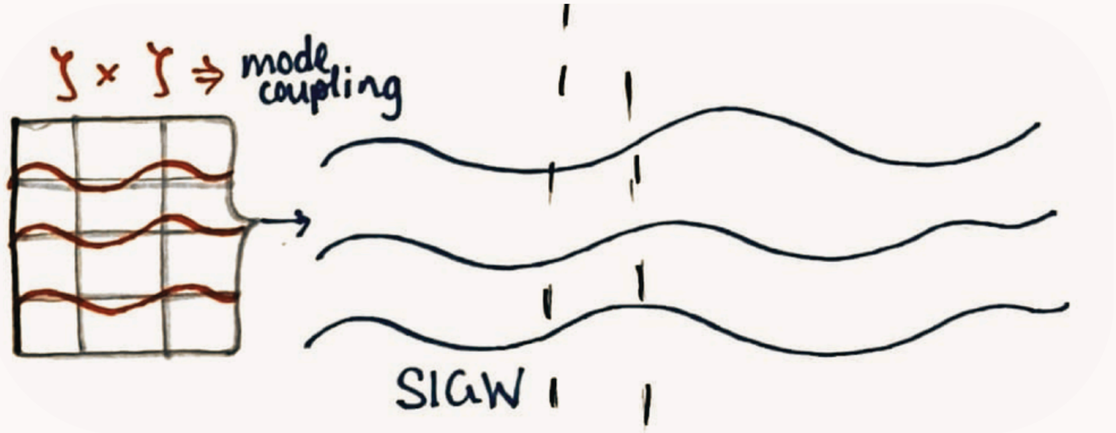
Tensor perturbations

GWs weakly interact with matter

Cosmological GWs come in a form of stochastic background due to quantum nature
Sourced by various mechanisms




References: arXiv:1811.08797




# Scalar - induced GWs

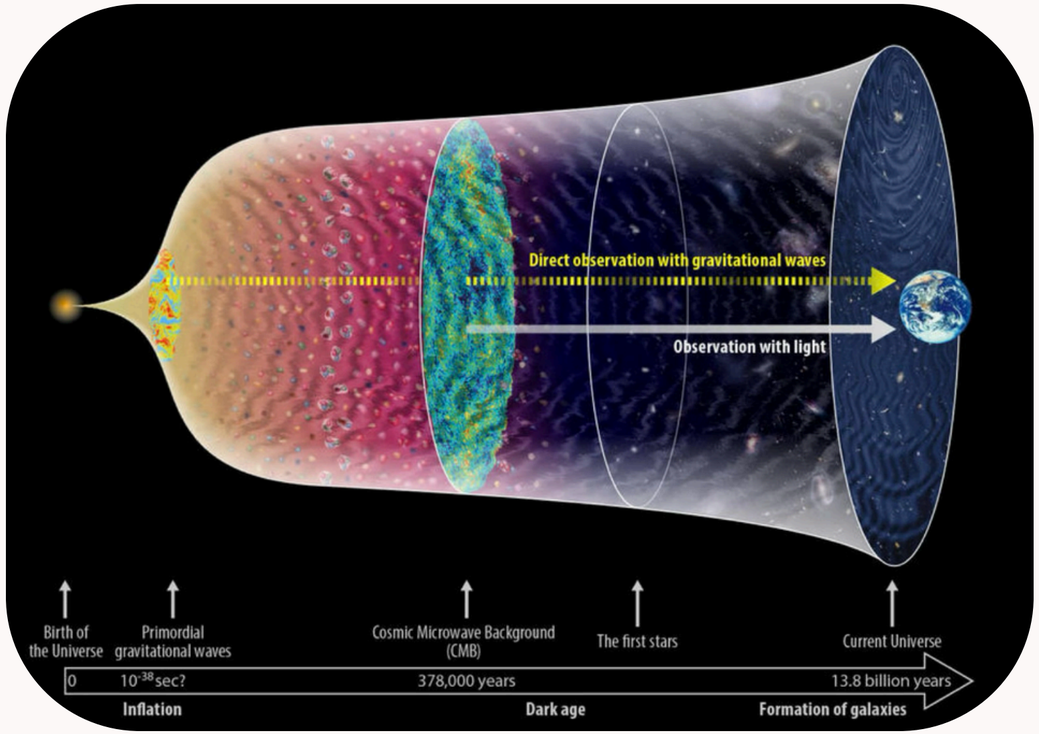
Secondary tensor modes produced due to the coupling of first-order scalar fluctuations

SIGWs trace small-scale primordial fluctuations, first-order scalar spectrum generating second-order GWs

# Why else do we want to study SIGWs?

# Detection

Characteristic frequencies fall in the band of current and future GW interferometers




Measurements of CMB and LSS established existence of scalar spectrum

Pulsar Timing Arrays (PTA) observation providing strong evidence for a SGWB & compatibility with “scalar-induced” background

# Detection

Pulsar Timing Arrays (PTA) observation interpretation

Performing Bayesian analysis to determine what model is favored

Criteria to see best fit – Bayes factor

Depends on modeling assumptions highlighting that more data and analysis are needed to discern between cosmological or astrophysical origin

Figueroa et al. 2023 (arXiv:2307.02399)

| Template | $\mathrm{BF}_{\mathrm{NANO}}$ | $\mathrm{BF}_{\mathrm{EPTA}}$ | $\mathrm{BF}_{\mathrm{comb}}$ | $\Delta\mathrm{AIC}_{\mathrm{comb}}$ |
|---|---|---|---|---|
| $\mathbf{PL}(n_t = 0)$ | 0.02 | 0.62 | 0.0014 | 10.24 |
| $\mathbf{PL}(n_t = 2/3)$ | 2.6 | 3.1 | 1.9 | -5.20 |
| $\mathbf{PL}(n_t, \mathcal{A}_*)$ | 16 | 4.1 | 180 | -12.25 |
| **field th. CS** | 0.18 | 1.3 | 0.019 | 5.22 |
| **super CS** | 29 | 3.3 | 58 | -11.11 |
| **gSIGWB** | 120 | 15 | 1300 | -13.54 |
| **ngSIGWB** | 66 | 8.7 | 780 | -6.82 |
| **AA** | 36 | 2.7 | 130 | -12.36 |
| **BPL** | 17 | 1.6 | 120 | -0.19 |
| **PhTNR** | 8.5 | 8.1 | 150 | -12.85 |
| **PhTRV** | 37 | 13 | 110 | -12.34 |
| **MHD** | 26 | 3.9 | 210 | -8.77 |

TABLE I. Bayes factors (BF) for NANOGrav, EPTA, and combined data sets, and Akaike Information Criterion (AIC) for the combined data sets.

# Primordial Black Holes

Which density fluctuations are the source of induced GWs?

# Primordial Black Holes

Large fluctuations re-entering horizon can collapse and form PBHs

After horizon re-entry, same scalar fluctuations oscillate with large amplitude and source the second-order tensor modes



PBH formation should be accompanied by production of SIGW, i.e. common origin

# Primordial Black Holes

After formation, gas of PBHs can be treated as fluid with density fluctuations

PBHs randomly distributed and if initial abundance is large, can dominated the universe content before evaporating



Gravitational waves produced via the gravitational potential of gas of PBHs

# Primordial Black Holes

PBH & SIGWs fluctuation common origin



SIGWs sourced by PBH gravitational potential



SIGWs can provide constraints on abundance of PBHs

References: arXiv:1612.06264v2 & arXiv:2012.08151v2

# Framework

# SIGW Equation of Motion

Tensor perturbations

source – couplings of scalar perturbations

$$h^{i(2)''}{}_j + 2\mathcal{H}h^{i(2)'}{}_j - \nabla^2 h^{i(2)}{}_j = -4\mathcal{P}^{li}_{jm}S^m_l .$$

For more references:

Matarrese, Mollerach and Bruni, 1997 (arXiv:astro-ph/9707278)

Andanda, Clarkson and Wands, 2007 (arXiv:gr-qc/0612013)

Baulmann et al. 2007 (arXiv:heb-th/0703290)

# SIGW Source Term

$$
\begin{aligned}
\mathcal{P}^{li}_{jm} S^{m}_{l} = {} & \partial^i \phi^{(1)} \partial_j \phi^{(1)} + 2\phi^{(1)} \partial^i \partial_j \phi^{(1)} - 2\psi^{(1)} \partial^i \partial_j \phi^{(1)} - \partial_j \phi^{(1)} \partial^i \psi^{(1)} - \partial^i \phi^{(1)} \partial_j \psi^{(1)} \\
& + 3\partial^i \psi^{(1)} \partial_j \psi^{(1)} + 4\psi^{(1)} \partial^i \partial_j \psi^{(1)} \\
& - \frac{4}{3\mathcal{H}^2 (1+w)} \left[ \partial^i \left( \psi^{(1)\prime} + \mathcal{H}\phi^{(1)} \right) \partial_j \left( \psi^{(1)\prime} + \mathcal{H}\phi^{(1)} \right) \right] \\
& + 4\psi^{(1)} \delta^{ik} \left[ \left( \partial_j \partial_k - \frac{1}{3} \nabla^2 \delta_{jk} \right) \left( \phi^{(1)} - \psi^{(1)} \right) \right] \quad .
\end{aligned}
$$

# Anisotropic stress

Standard GR

Contribution from free-streaming photon and neutrinos is small and negligible at late times

At linear-order, scalar potentials are set to be equal to one another

$$\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta^i_j\right)\left(\phi^{(1)} - \psi^{(1)}\right) = \kappa^2 \pi^{(1)i}_j .$$



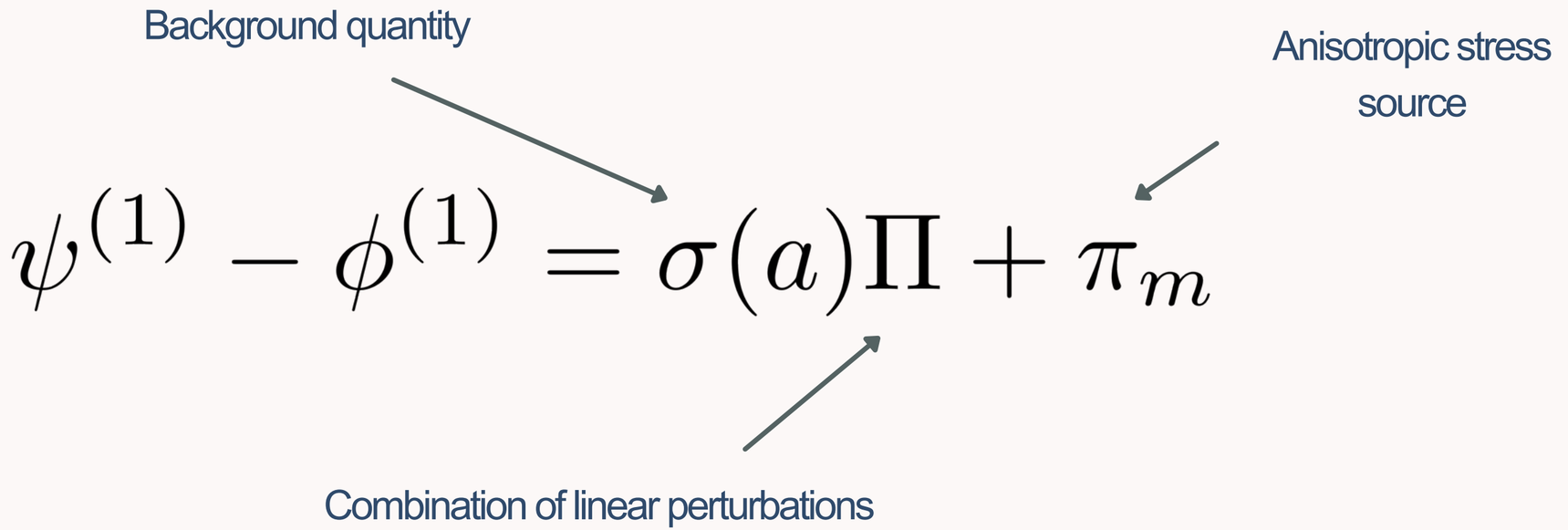
$$\psi^{(1)} = \phi^{(1)}$$



# Mistmatch between scalar potentials

# Gravitational slip

A non-standard relation between the potentials can arise due to modification in geometric part of Einstein equation




Can lead to non-standard propagation of tensor modes

# Beyond GR theories

- Lagrangian density, f, is a function of Ricci scalar

$$F(R)G_{\mu}^{\lambda} = g^{\lambda\nu}g_{\mu\nu}\frac{(f(R) - RF(R))}{2} + g^{\lambda\nu}\nabla_{\mu}\nabla_{\nu}F(R) - g^{\lambda\nu}g_{\mu\nu}\Box F(R) + \kappa^2 T_{\mu}^{\lambda}$$

- Model considered:

$$f(R) = R + \alpha R^2$$

- Concentrating on the first-order corrections

# Gravitational slip

Combination of linear perturbations

$$\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta^i{}_j\right)\left(\psi^{(1)} - \phi^{(1)} - \frac{F^{(1)}}{\bar{F}}\right) = \frac{1}{\bar{F}}\kappa^2\pi^{i(1)}{}_j$$

Background quantity

Anisotropic stress source

# Observable

$$\Omega_{\rm GW}(k,\tau) = \frac{1}{48}\left(\frac{k}{a(\tau)H(\tau)}\right)^2 \sum_{\lambda=+,\times} \overline{\Delta^2_{h,\lambda}(k,\tau)}$$

$$\Omega_{\rm GW}(k,\tau) = \Omega_{\rm GW,GR}(k,\tau) + \alpha\,\delta\Omega_{\rm GW}(k,\tau)$$

First-order correction

# SIGW Equation of Motion

$$h_\lambda^{(2)\prime\prime}(\mathbf{k},\tau) + \left(2\mathcal{H} + \frac{\bar{F}_\mathbf{k}'}{\bar{F}_\mathbf{k}}\right) h_\lambda^{(2)\prime}(\mathbf{k},\tau) + k^2 h_\lambda^{(2)}(\mathbf{k},\tau) = S_\lambda(\mathbf{k},\tau)$$

$$= S_{\lambda,\mathrm{GR}}(\mathbf{k},\tau) + \alpha\delta S_\lambda(\mathbf{k},\tau)$$

# GR spectrum: $\Omega_{\mathrm{GW,GR}}(k,\tau)$

$$h_\lambda^{(2)\prime\prime}(\mathbf{k},\tau) + \left(2\mathcal{H} + \cancel{\frac{\bar{F}'_{\mathbf{k}}}{\bar{F}_{\mathbf{k}}}}\right) h_\lambda^{(2)\prime}(\mathbf{k},\tau) + k^2 h_\lambda^{(2)}(\mathbf{k},\tau) = S_\lambda(\mathbf{k},\tau)$$

$$= S_{\lambda,\mathrm{GR}}(\mathbf{k},\tau)$$

Using GR solution for scalar potential

$$\phi_{\mathrm{GR}}^{(1)} \times \phi_{\mathrm{GR}}^{(1)}$$

# Modifed spectrum: $\alpha\,\delta\Omega_{\rm GW}(k,\tau)$

$$h_\lambda^{(2)\prime\prime}(\mathbf{k},\tau)+\left(2\mathcal{H}+\frac{\bar{F}_\mathbf{k}'}{\bar{F}_\mathbf{k}}\right)h_\lambda^{(2)\prime}(\mathbf{k},\tau)+k^2h_\lambda^{(2)}(\mathbf{k},\tau)=S_\lambda(\mathbf{k},\tau)$$

$$=S_{\lambda,\rm GR}(\mathbf{k},\tau)+\alpha\delta S_\lambda(\mathbf{k},\tau)$$

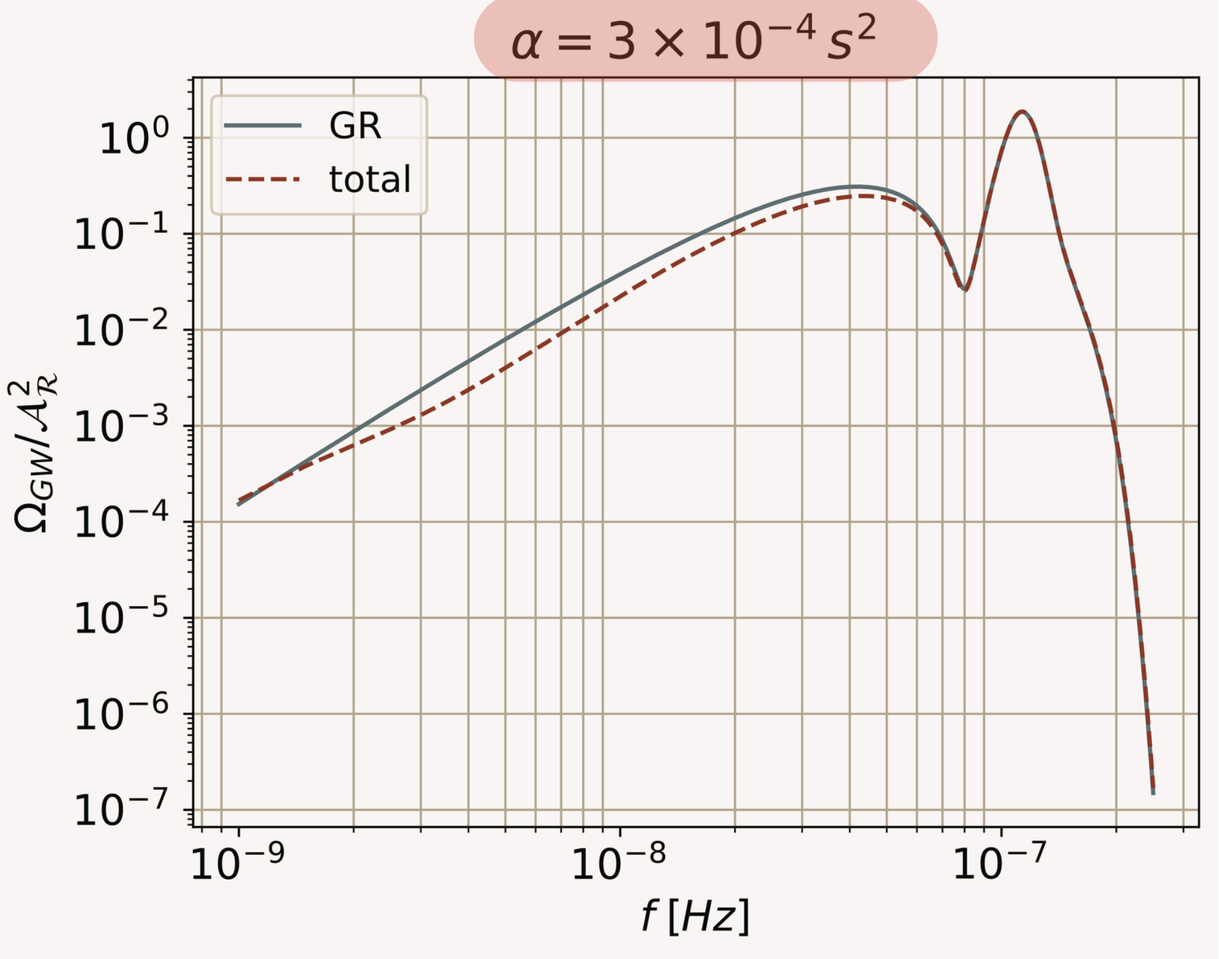
Using first-order correction for scalar potential
i.e. purely from gravitational slip

$\delta\phi^{(1)}$ ✕ $\phi^{(1)}_{\rm GR}$

Using GR solution for scalar potential
i.e. modifications from Einstein-Hilbert action

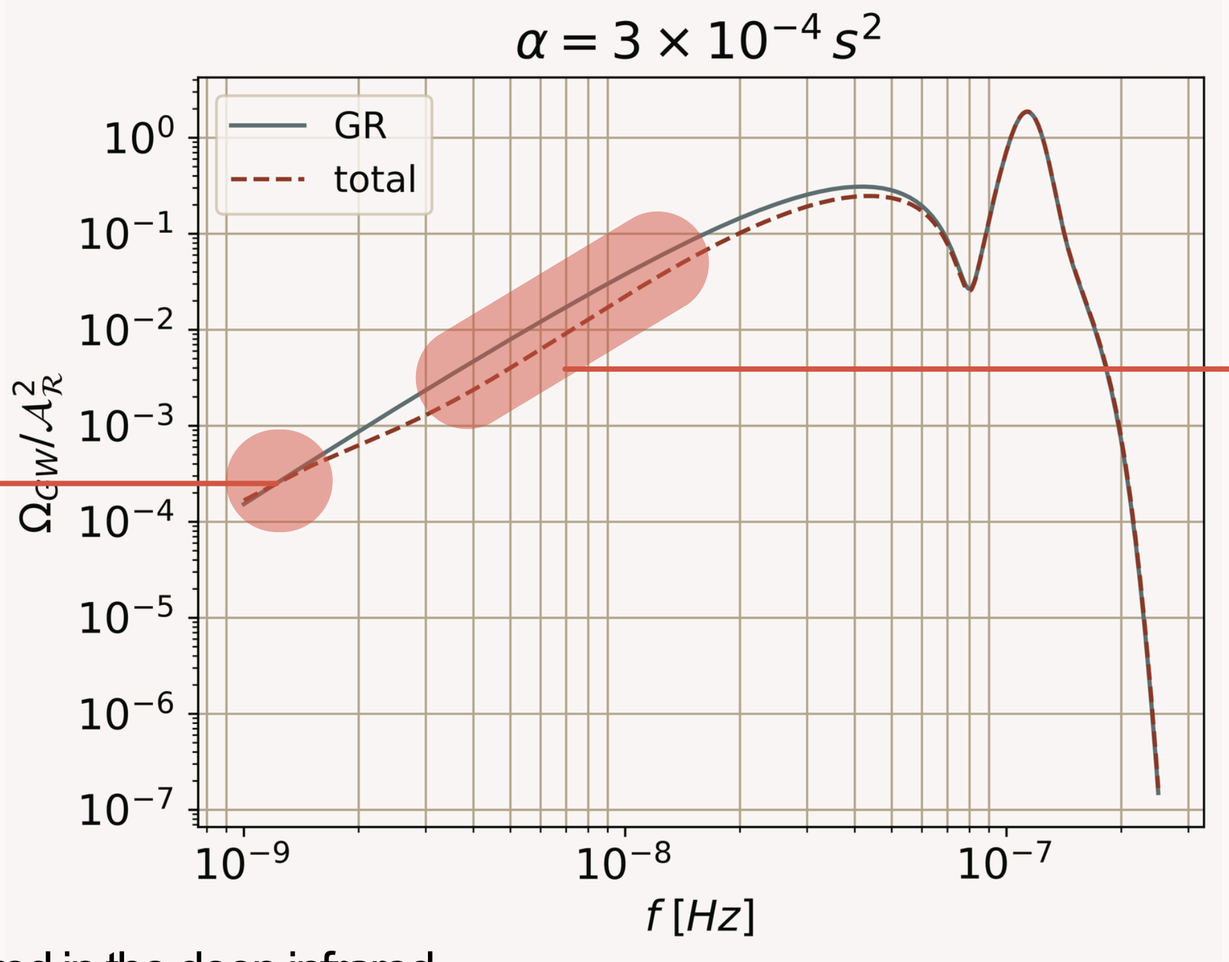
$\phi^{(1)}_{\rm GR}$ ✕ $\phi^{(1)}_{\rm GR,m}$

# Observable - PTA




AK, MT, AM, SM & AR, 2025 — arXiv:2502.20137

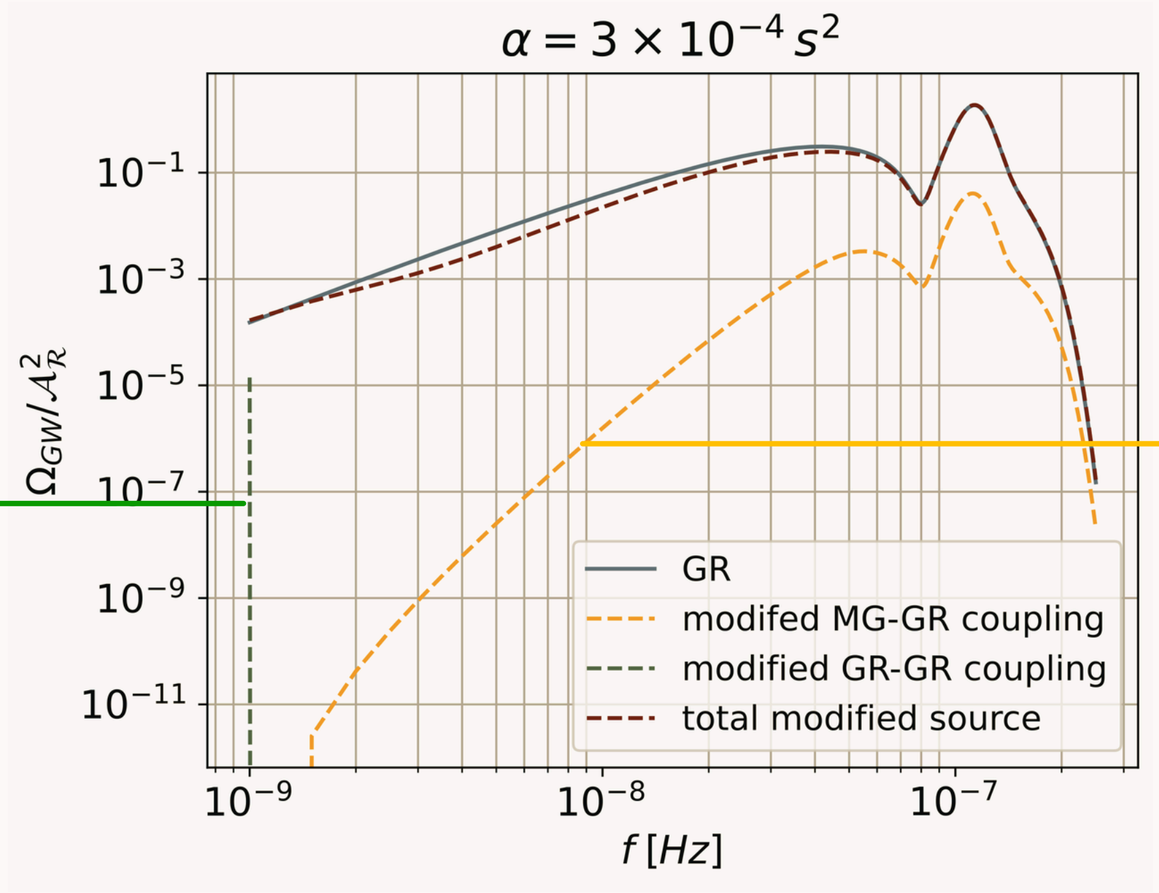
# Observable




Spectrum with modification acts as a damping term – supression of the signal amplitude

GR solution recovered in the deep infrared regime

# Observable




second-order modification of Einstein-Hilbert action

the gravitational slip contribution – dominant

# Open questions?

1

Validity of the model?

↓

Extensions to other theories?

2

Pre-factor constraints?

3

Bound on $\alpha$ ?

# Take home message

**Scalar-induced gravitational waves can open-up a new door for cosmology**

- Detectability
- Probing early-universe models and theories
- Constraints on PBH abundance

# Back up slides

# Gravitational slip

$$\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta^i{}_j\right)\left(\psi^{(1)} - \phi^{(1)} - \frac{F^{(1)}}{\bar{F}}\right) = \frac{1}{\bar{F}}\kappa^2\pi^{i(1)}{}_j$$

$$\psi^{(1)} - \phi^{(1)} = \frac{F^{(1)}}{\bar{F}}$$

# Gravitational slip

Decomposing the first order scalar perturbations with first order correction

$$\phi^{(1)} = \phi^{(1)}_{gr} + \alpha\delta\phi^{(1)}$$

$$\psi^{(1)} = \psi^{(1)}_{gr} + \alpha\delta\psi^{(1)}$$

$$\downarrow$$

$$\phi^{(1)}_{gr}$$

$$\psi^{(1)} - \phi^{(1)} = \frac{F^{(1)}}{\bar{F}}$$

$$\downarrow$$

$$\delta\psi^{(1)} = \delta\phi^{(1)} + 2a^{-2}\left(-6\phi^{(1)\prime\prime}_{gr} - 24\mathcal{H}\phi^{(1)\prime}_{gr} - 12\left[\mathcal{H}' + \mathcal{H}^2\right]\phi^{(1)}_{gr} + 2\nabla^2\phi^{(1)}_{gr}\right)$$

# First-order correction

$$\delta\phi^{(1)\prime\prime} + 4\tau^{-1}\delta\phi^{(1)\prime} + \frac{1}{3}k^2\delta\phi^{(1)} = \left(a_*\frac{\tau}{\tau_*}\right)^{-2}\left[\left(\frac{2}{3}k^4 - 4\tau^{-2}k^2\right)\phi^{(1)}_{\rm GR} + 8\tau^{-1}k^2\phi^{(1)\prime}_{\rm GR} + (4k^2 - 60\tau^{-2})\phi^{(1)\prime\prime}_{\rm GR} + 24\tau^{-1}\phi^{(1)\prime\prime\prime}_{\rm GR} + 6\phi^{(1)\prime\prime\prime\prime}_{\rm GR}\right].$$

GR scalar potential solution evaluated in RD era with CDM correction

# Key ingredients

**GR solution in pure RD epoch**

$$\phi^{(1)}_{\mathrm{GR}}$$

**GR solution with CDM correction in RD epoch**

$$\phi^{(1)}_{\mathrm{GR,m}}$$

**Modified potential**

$$\delta\phi^{(1)}$$

# Roadmap

Solving second-order E.o.M

+

Analytical and numerical source function

+

Determine pre-factor

$$\overline{\Delta^2_{h,\lambda}(k,\tau)}$$

$$\frac{\delta\Omega_{\mathrm{GW}}(k,\tau)}{\Omega_{\mathrm{GW,GR}}(k,\tau)}$$

# Power spectrum

$$\Delta_h^2(k,\tau) = 8\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1-u^2+v^2)^2}{4uv}\right)^2 \times \left(I^2_{\mathrm{GR,RD}}(v,u,x) + 2\alpha\delta I_{\mathrm{RD}}(v,u,x) I_{\mathrm{GR,RD}}(v,u,x)\right) \Delta_\zeta^2(ku)\Delta_\zeta^2(kv)$$

Dependence on primordial curvature power spectrum

$$\Delta_\zeta^2(k) = \frac{\mathcal{A}_\zeta}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\ln^2(k/k_*)}{2\sigma^2}\right)$$

# Power spectrum

$$\Delta_h^2(k,\tau) = 8\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1-u^2+v^2)^2}{4uv}\right)^2 \times \left(I^2_{\mathrm{GR,RD}}(v,u,x) + 2\alpha\delta I_{\mathrm{RD}}(v,u,x) I_{\mathrm{GR,RD}}(v,u,x)\right) \Delta_\zeta^2(ku)\Delta_\zeta^2(kv)$$

Integration kernel which includes the convolution of the source term in Fourier space

# Modified Source Function

$$\delta f(|\mathbf{k}-\mathbf{q}|,q,\tau) = \frac{4}{9}\Big\{3T_{\delta\phi}(q\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + 3T_{\phi}(q\tau)T_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+\tau T'_{\delta\phi}(q\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + \tau T_{\delta\phi}(q\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+\tau T'_{\phi}(q\tau)T_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau) + \tau T_{\phi}(q\tau)T'_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+\tau^2 T'_{\phi}(q\tau)T'_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau) + \tau^2 T'_{\delta\phi}(q\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$-\left(\frac{\tau_*}{a_*}\right)^2 \tau^{-2}\left[-96T'_{\phi}(q\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)\right.$$
$$+30\left(T_{\phi}(\mathbf{q}\tau)T''_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + T''_{\phi}(\mathbf{q}\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)\right)$$
$$+6\tau\left(T_{\phi}(\mathbf{q}\tau)T'''_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + T'''_{\phi}(\mathbf{q}\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)\right)$$
$$+18\tau\left(T'_{\phi}(q\tau)T''_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + T''_{\phi}(\mathbf{q}\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)\right)$$
$$+6\tau^2\left(T'_{\phi}(q\tau)T'''_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + T'''_{\phi}(\mathbf{q}\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)\right)$$
$$+2\left(\mathbf{q}^2 + |k-\mathbf{q}|^2\right)T_{\phi}(\mathbf{q}\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+2\tau^2\left(\mathbf{q}^2 + |k-\mathbf{q}|^2\right)T'_{\phi}(q\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+2\tau\mathbf{q}^2\left(T'_{\phi}(q\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) - T_{\phi}(\mathbf{q}\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)\right)$$
$$\left.+2\tau|k-\mathbf{q}|^2\left(T_{\phi}(\mathbf{q}\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) - T'_{\phi}(q\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)\right)\right]\Big\}\,.$$

the gravitational slip

$S_{\lambda,\mathrm{GR}}(\mathbf{k},\tau)$

$\delta\phi^{(1)} \times \phi^{(1)}_{\mathrm{GR}}$

second-order modification

$\delta S_{\lambda}(\mathbf{k},\tau)$

$\phi^{(1)}_{\mathrm{GR}} \times \phi^{(1)}_{\mathrm{GR,m}}$

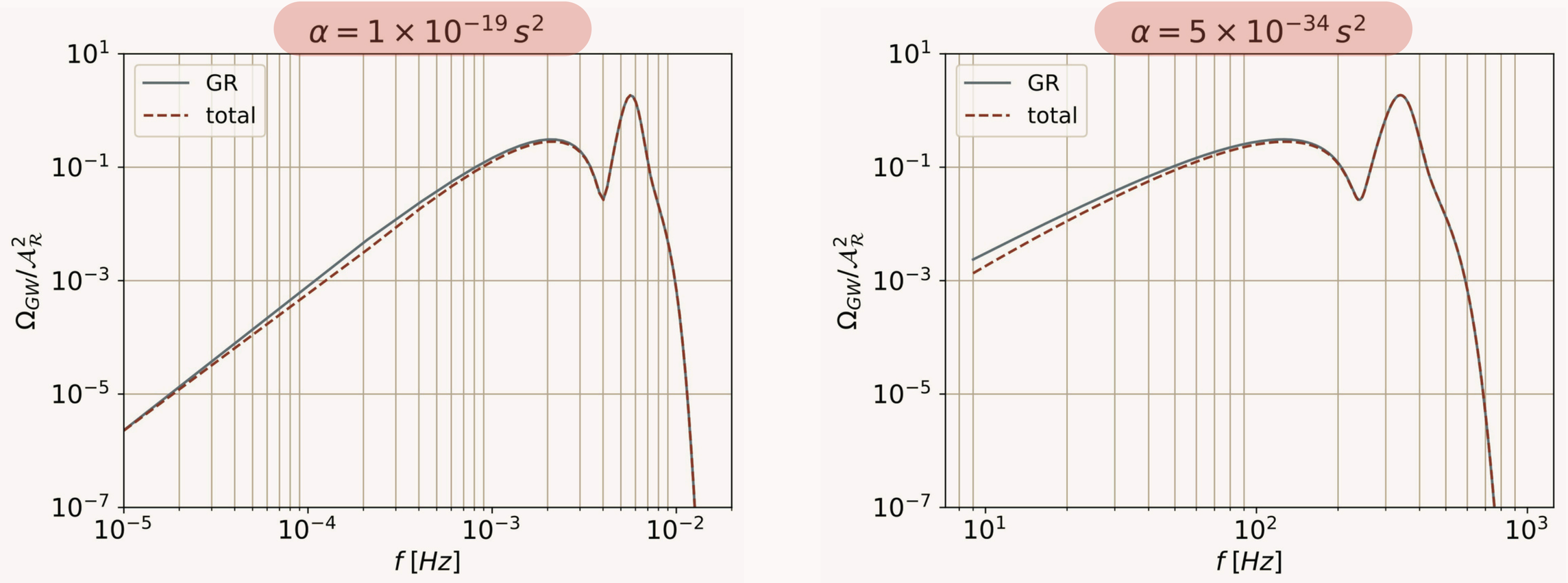
# Pre-factor of ratio

$$\frac{\delta\Omega_{\mathrm{GW}}(k,\tau)}{\Omega_{\mathrm{GW,GR}}(k,\tau)} \propto \alpha \left(\frac{\tau^*}{a^*}\right)^2 \frac{1}{\tau_{eq}} k^3$$

Assuming a value while remaining perturbative and has to be positive, due to ‘scalaron’ mass

Constrain by using the background Friedmann equation deep in the radiation era

# Observable - LISA & LVK




AK, MT, AM, SM & AR, 2025 — arXiv:2502.20137

# Gravitational slip contribution to $\delta\Omega_{\rm GW}(k,\tau)$

$$\delta f(|\mathbf{k}-\mathbf{q}|,q,\tau) = \frac{4}{9}\Big\{3T_{\delta\phi}(q\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + 3T_{\phi}(q\tau)T_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+ \tau T'_{\delta\phi}(q\tau)T_{\phi}(|\mathbf{k}-\mathbf{q}|\tau) + \tau T_{\delta\phi}(q\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+ \tau T'_{\phi}(q\tau)T_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau) + \tau T_{\phi}(q\tau)T'_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$
$$+ \tau^2 T'_{\phi}(q\tau)T'_{\delta\phi}(|\mathbf{k}-\mathbf{q}|\tau) + \tau^2 T'_{\delta\phi}(q\tau)T'_{\phi}(|\mathbf{k}-\mathbf{q}|\tau)$$

the gravitational slip

$S_{\lambda,\rm GR}(\mathbf{k},\tau)$

$\delta\phi^{(1)} \times \phi^{(1)}_{\rm GR}$

# Contribution from action modifications to $\delta\Omega_{\rm GW}(k,\tau)$

$$
\begin{aligned}
\delta f(|\mathbf{k}-\mathbf{q}|,q,\tau) = & -\left(\frac{\tau_*}{a_*}\right)^2 \tau^{-2}\left[-96T'_\phi(q\tau)T'_\phi(|\mathbf{k}-\mathbf{q}|\tau)\right. \\
& +30\left(T_\phi(\mathbf{q}\tau)T''_\phi(|\mathbf{k}-\mathbf{q}|\tau)+T''_\phi(\mathbf{q}\tau)T_\phi(|\mathbf{k}-\mathbf{q}|\tau)\right) \\
& +6\tau\left(T_\phi(\mathbf{q}\tau)T'''_\phi(|\mathbf{k}-\mathbf{q}|\tau)+T'''_\phi(\mathbf{q}\tau)T_\phi(|\mathbf{k}-\mathbf{q}|\tau)\right) \\
& +18\tau\left(T'_\phi(q\tau)T''_\phi(|\mathbf{k}-\mathbf{q}|\tau)+T''_\phi(\mathbf{q}\tau)T'_\phi(|\mathbf{k}-\mathbf{q}|\tau)\right) \\
& +6\tau^2\left(T'_\phi(q\tau)T'''_\phi(|\mathbf{k}-\mathbf{q}|\tau)+T'''_\phi(\mathbf{q}\tau)T'_\phi(|\mathbf{k}-\mathbf{q}|\tau)\right) \\
& +2\left(\mathbf{q}^2+|k-\mathbf{q}|^2\right)T_\phi(\mathbf{q}\tau)T_\phi(|\mathbf{k}-\mathbf{q}|\tau) \\
& +2\tau^2\left(\mathbf{q}^2+|k-\mathbf{q}|^2\right)T'_\phi(q\tau)T'_\phi(|\mathbf{k}-\mathbf{q}|\tau) \\
& +2\tau\mathbf{q}^2\left(T'_\phi(q\tau)T_\phi(|\mathbf{k}-\mathbf{q}|\tau)-T_\phi(\mathbf{q}\tau)T'_\phi(|\mathbf{k}-\mathbf{q}|\tau)\right) \\
& \left.+2\tau|k-\mathbf{q}|^2\left(T_\phi(\mathbf{q}\tau)T'_\phi(|\mathbf{k}-\mathbf{q}|\tau)-T'_\phi(q\tau)T_\phi(|\mathbf{k}-\mathbf{q}|\tau)\right)\right]\Big\}\,.
\end{aligned}
$$

second-order modification

$\delta S_\lambda(\mathbf{k},\tau)$

$\phi^{(1)}_{\rm GR} \times \phi^{(1)}_{\rm GR,m}$