Quasi-local masses and cosmological coupling of astrophysical compact objects



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mainly based on:

- Cadoni, Sanna, Pitzalis, Banerjee, **RM**, Hazra, Branchesi
  JCAP 11 (2023) 007; **2306.11588**
- Cadoni, **RM**, Pitzalis, Sanna
  JCAP 03 (2024) 026; **2309.16444**

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One question

Within General Relativity (GR) can we consistently describe the dynamics of local matter inhomogeneities embedded in an expanding universe and their interplay with the large-scale dynamics?

One question…

are local gravitational systems affected by the large-scale cosmological dynamics?.

..or more

are local gravitational systems affected by the large-scale cosmological dynamics?.

Focus of this Talk

is the large-scale cosmological dynamics influenced by local matter inhomogeneities?

..or more

are local gravitational systems affected by the large-scale cosmological dynamics?

Focus of this Talk

is the large-scale cosmological dynamics influenced by local matter inhomogeneities?

but stay tuned on this!

what is the homogeneity and isotropy scale at late times?

Local and non-local: MS vs ADM mass

Arnowitt Deser Misner (ADM) mass

- Vacuum solution;
- Non-local quantity, as it is defined at the boundary of spacetime;
- Cannot be defined in non-asymptocally flat spacetimes as those corresponding to cosmologically embedded objects.

Misner Sharp (MS) mass

- Suitable for objects not in vacuum, as non-singular black holes;
- Quasi-local quantity, related to the astrophysical properties of virialized systems;
- Covariantly defined also in non-asymptocally flat spacetimes.

$$M_{\mathrm{MS}}(r,t) = \frac{\ell}{2G}\left(1 - \nabla_a \ell \, \nabla^a \ell\right)$$

$\ell = ar$

Do local gravitational systems couple to the large-scale cosmological dynamics?

- Cosmological expansion does not affect small-scale Newtonian systems

What about relativistic bodies such as black holes (BHs)?

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What about relativistic bodies such as black holes (BHs)?

- The first attempt to address the question is done by embedding a **Schwarzschild BH in a FLRW background**:

$$ds^2 = -\frac{\left(1-\frac{Gm(t)}{2r}\right)^2}{\left(1+\frac{Gm(t)}{2r}\right)^2}dt^2 + a^2\left(1+\frac{Gm(t)}{2r}\right)^4\left(dr^2+r^2d\Omega^2\right),$$

McVittie (1933)

Common arguments against cosmological coupling

- Cosmological coupling does not occur to any solution of Einstein's equations embedding black holes in expanding universes, *e.g.* McVittie

Gaur&Visser 2308.07374

- Huge separation of scales between BH physics and cosmological dynamics
  - Heaviest known BH $\longrightarrow$ ~ 10^{-3} parsec
  - Homogeneity and isotropy scale $\longrightarrow$ ~ 10^{8} parsec

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Our proposal: compact objects sourced by anisotropic fluids

- We describe the source of gravitational field with an **anisotropic fluid**

$$T_{\mu\nu} = (\rho + p_\perp)\, u_\mu u_\nu + p_\perp\, g_{\mu\nu} - (p_\perp - p_\parallel)\, w_\mu w_\nu \,,$$

- u_μ: timelike vector ($g^{\mu\nu} u_\mu u_\nu = -1$).
- w_μ: spacelike vector ($g^{\mu\nu} w_\mu w_\nu = 1$).
- $u_\mu w^\mu = 0$.
- ρ: energy density
- $p_\parallel$: radial pressure
- $p_\perp$: transverse pressure

- The spacetime is parametrized by

$$\mathrm{d}s^2 = a^2(\eta)\left[-e^{\alpha(\eta,r)}\mathrm{d}t^2 + e^{\beta(\eta,r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right] .$$

- η: conformal time.
- $a(\eta)$: the scale factor.
- $\alpha(\eta, r)$ and $\beta(\eta, r)$: the metric functions.

Retrieving FLRW cosmology implies choosing models that satisfies

- $\alpha \to 0$
- $e^\beta \to 1/(1 - \mathcal{K} r^2)$

Our proposal: the Misner-Sharp (MS) mass

Density profile of the object:

$$8\pi G\rho = \frac{8\pi G}{3}\rho_1 \frac{1}{r^2}\partial_r\left(r^3 e^{-\alpha}\right) + \frac{1}{a^2 r^2}\partial_r(r - re^{-\beta})\,.$$

- **Curvature term responsible for the coupling**

- Purely cosmological contribution.
- Not relevant at scale L.

- **Model-dependent correction (subleading term)**

$$M(\eta) = 4\pi a^3(\eta)\int_0^L \mathrm{d}r\, r^2\,\rho(r,\eta) = \frac{4\pi}{3}\rho_1 a^3 L^3\, e^{-\alpha(L)} + M(a_i)\frac{a}{a_i}\left[1 - e^{-\beta_0(L)}a^{k_L}\right],$$

- Universal cosmological Schwarzschild mass: $\frac{a_i L}{2\,G}$.

Our prediction: linear mass growth of the MS mass

- The equation is valid for any compact *regular* object sourced by anisotropic fluid allowed by General Relativity.

$$M(a) = M(a_i)\left(\frac{a}{a_i}\right)^k \,, \qquad \text{with} \quad a \geq a_i \,,$$

$$M(\eta) = 4\pi a^3(\eta)\int_0^L \mathrm{d}r\, r^2\, \rho(r,\eta) = \frac{4\pi}{3}\rho_1 a^3 L^3\, e^{-\alpha(L)} + M(a_i)\frac{a}{a_i}\left[1 - e^{-\beta_0(L)} a^{k_L}\right] \,,$$

- Length scale *L*:
  - For BHs it is the event horizon
  - For horizonless compact object it represents the radius enclosing 99% of the mass

We predict a cosmological coupling constant k = 1

MS mass for isotropic sources → no mass growth

- **McVittie solution: perfect, isotropic and spherically-symmetric fluid as a source embedded in a FLRW background**

McVittie (1933)

$$ds^2 = -\frac{\left(1-\frac{Gm(t)}{2r}\right)^2}{\left(1+\frac{Gm(t)}{2r}\right)^2}dt^2 + a^2\left(1+\frac{Gm(t)}{2r}\right)^4\left(dr^2+r^2d\Omega^2\right),$$

$$M_{\rm MS} = \frac{R}{2G}\left(\frac{2Gm_0}{R}+H^2R^2\right) = m_0 + \frac{H^2}{2G}R^3.$$

The first term is the ADM mass of the Schwarzschild BH. The second term is the mass contribution due to the constant cosmological density over a volume R^3.

MS mass for anisotropic sources → linear mass growth

- Compact objects sourced by anisotropic fluids:

$$M_{\rm MS} = \frac{ar}{2G}\left[1+\frac{\dot{a}^2}{a^2}r^2e^{-\alpha}-\frac{a^2}{a^2}e^{-\beta}\right]$$
$$= \frac{4\pi}{3}\rho_1 a^3 r^3 e^{-\alpha} + \frac{ar}{2G}\left[1-e^{-\beta_0(r)}a^{k(r)}\right],$$

- Cosmological coupling is present independently from the specific equation of state of the cosmological fluid
- The coupling is not due to any accretion flow, as we imposed the absence of radial fluxes

MS mass of black holes and cosmological coupling

Black hole event horizons are cosmologically coupled

2407.14549

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It is shown that an exactly static and spherically symmetric black hole event horizon cannot be embedded in a time-dependent geometry. Forcing it to do so results in a naked null singularity at the would-be horizon. Therefore, since the universe is expanding, black holes must couple to the cosmological expansion, which was suggested as the growth mechanism for supermassive black holes

MS mass of black holes and cosmological coupling

Cosmologically Coupled Black Holes with Regular Horizons

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2601.03296

We present the most general and exact solution of Einstein's gravity sourced by an anisotropic fluid describing the cosmological embedding (CE) of a static and spherically-symmetric object, including black holes (BHs) or exotic compact objects, without radial energy influx and in an arbitrary Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology. This is done fully considering backreaction of the local geometry on the cosmological dynamics. Our solution is free of curvature singularities at the would-be BH event horizon, thus solving a main issue of the CE of BH solutions proposed so far. As a byproduct, we derive a new CE of the Schwarzschild BH - distinct from McVitties's original proposal - that is regular everywhere except at the central singularity.

Take-home message on the theory

- The ADM mass is a non-local quantity, therefore unable to quantify local properties.
- The huge separation of scales cannot justify the use of the ADM mass as it cannot be defined in non-asymptotically flat spacetimes, nor for objects sourced by anisotropic fluids.
- Cosmological coupling occurs whenever the energy of the object is quantified by the MS, with zero coupling only when the MS mass is fully equivalent to the ADM mass.
- For singular BHs there is no mass growth due to cosmological coupling, whereas nonsingular compact objects couple to the cosmological background, as quantified by their MS mass.

Take-home message on the theory

$$M(z) = M(z_i)\left(\frac{1+z_i}{1+z}\right)^k$$

- $k = 0$ ⟶ nonsingular GR BHs incompatible with observations
- $k = 1$ ⟶ smoking gun that GR BHs are nonsingular
- k = other value ⟶ astrophysical BHs cannot be described with GR

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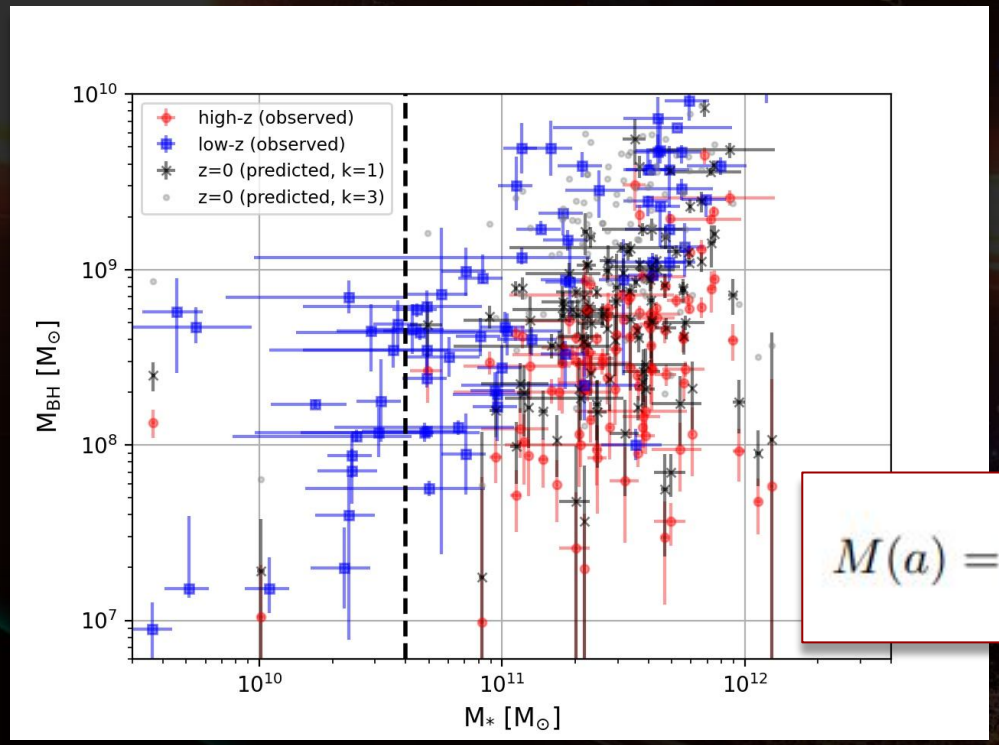
Can we set observational constraints on the cosmological coupling strength k ?

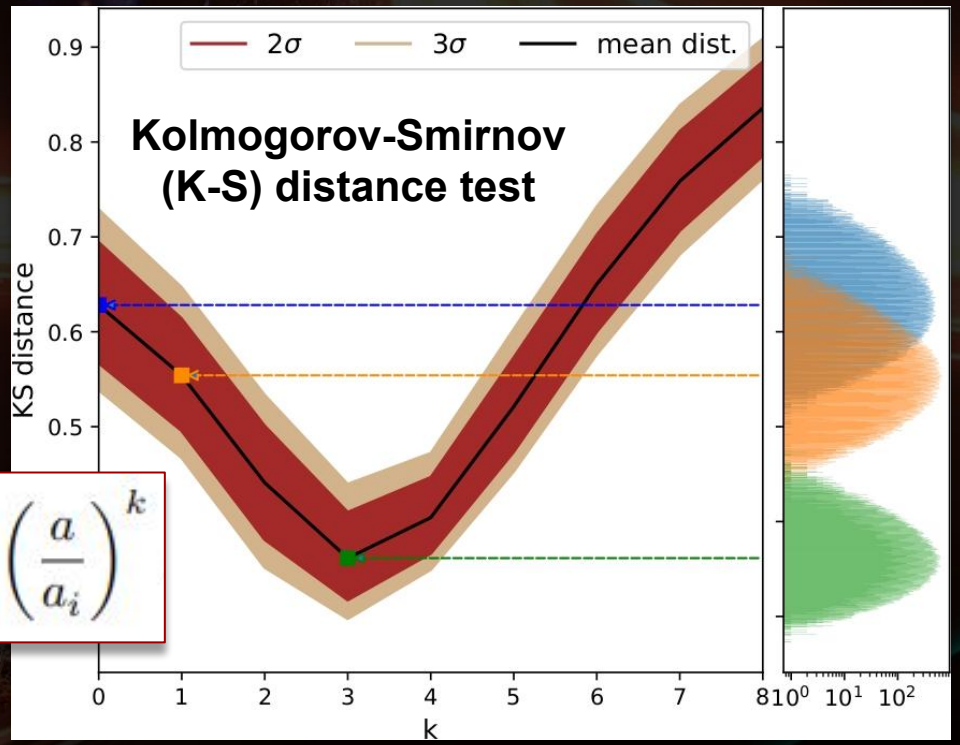
Constraints from super massive BHs (SMBHs)

Comparison between theoretical predictions and mass measurements of SMBHs in elliptical galaxies.

- Red sequence elliptical galaxies (SMBHs growth via accretion is negligible):
  - $0.0016 \leq z \leq 0.19$ (low redshift dataset, D. Farrah *et al.* The Astrophysical Journal 943, 133 (2023))
  - $0.8 \leq z \leq 0.9$ (high redshift dataset, cross matching WISE (R.S. Barrows *et al.* (2021)) + SDSS (S. Rakshit *et al.* (2020)) survey.)



$$M(a) = M(a_i)\left(\frac{a}{a_i}\right)^k$$



Constraints from GWs

- **Implementation within BBH population synthesis codes of the extra-term describing the adiabatic decay due to cosmological coupling** cosmoRATE, B-POP

$$M(z) = M(z_i)\left(\frac{1+z_i}{1+z}\right)^k$$

$$\frac{\mathrm{d}R}{\mathrm{d}a} = \underbrace{\frac{\partial R}{\partial L}\frac{\mathrm{d}L}{\mathrm{d}a}\bigg|_{e,M} + \frac{\partial R}{\partial e}\frac{\mathrm{d}e}{\mathrm{d}a}\bigg|_{L,M}}_{\text{Radiative decay}} + \underbrace{\frac{\partial R}{\partial M}\frac{\mathrm{d}M}{\mathrm{d}a}\bigg|_{e,L}}_{\text{Adiabatic decay}} .$$

- **Full hierarchical Bayesian analyses on GW data** ICAROGW

Synthetic catalogs
Seven Universes: masses m_1, m_2 and merger redshift z for every BBH

Normalizing flows
A neural density estimator learns $p(m_1, m_2, z)$ per channel, up to z = 2.5

Hierarchical inference
icarogw fits an isolated + cluster mixture, with dynamical fraction φ and H_0 free

Model comparison
Bayesian evidence by nested sampling, plus expected detections N_{exp} vs 153 observed

Constraints from GWs

ONGOING WORK led by M. Pitzalis, I. Usai (UniCa), A. Scarpa (INFN Rome)

$$\dot{R} / R = 2R \langle \dot{E}_{GW} \rangle / (G\,\mu(t)\,M(t)) - 3k\,H(t)$$

$$t_{GW}^{std} \propto \frac{1}{\mu M^2}$$

Assumptions

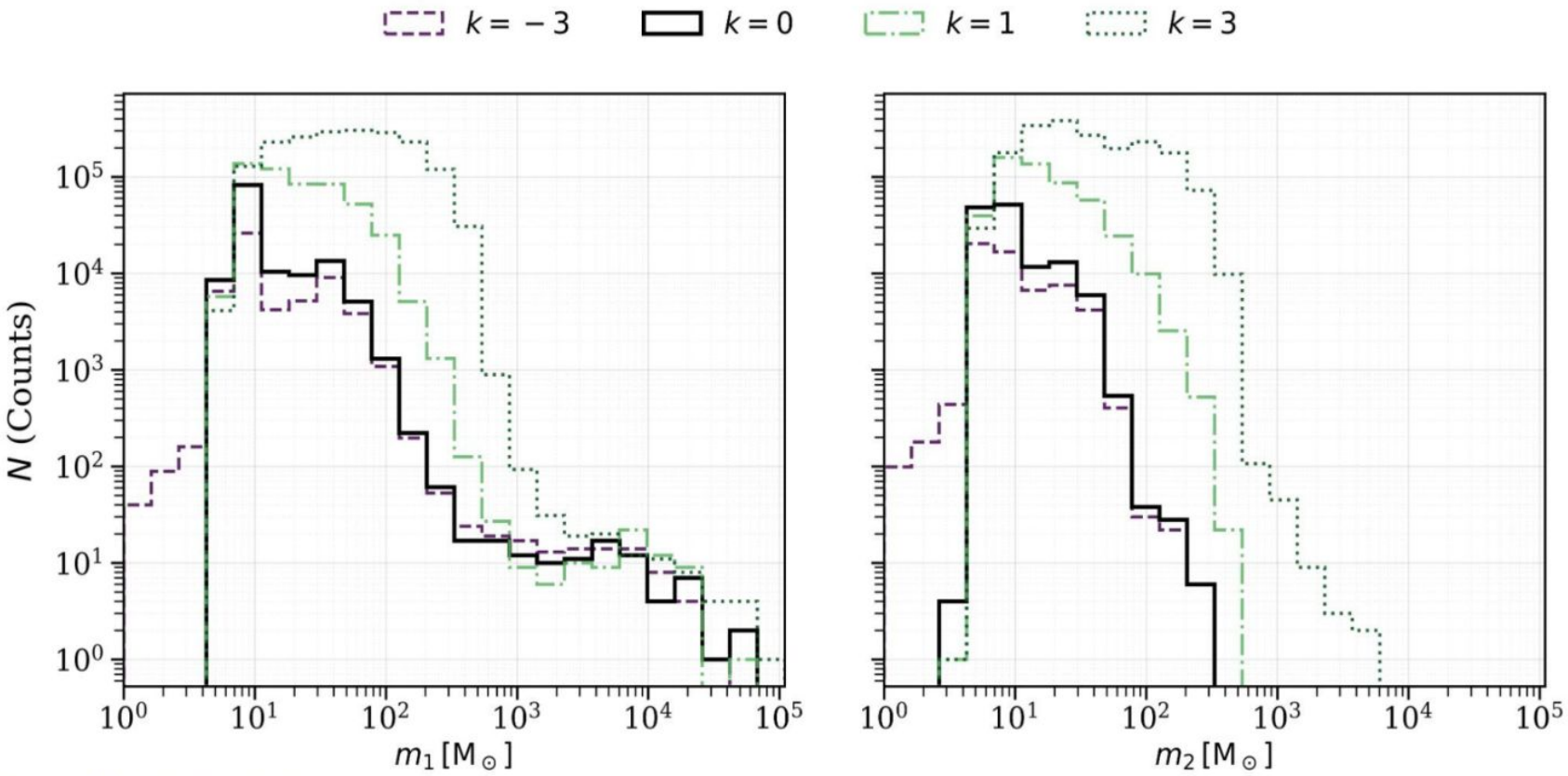
- **Adiabatic:** the masses change slowly compare to the orbital period, so the binary stays Keplerian at every instant

What each term does

- **GW term:** standard Peters decay, but with $m_1(t)$, $m_2(t)$ growing or shrinking as a^k
- **$-3kH(t)$:** mass change alone drifts the orbit; it shrinks for $k > 0$, widens for $k < 0$

Net effect for k > 0: heavier BHs radiate faster, so binaries that would never merge in a Hubble time now do. $k < 0$ does the opposite.

Constraints from GWTC-4 data

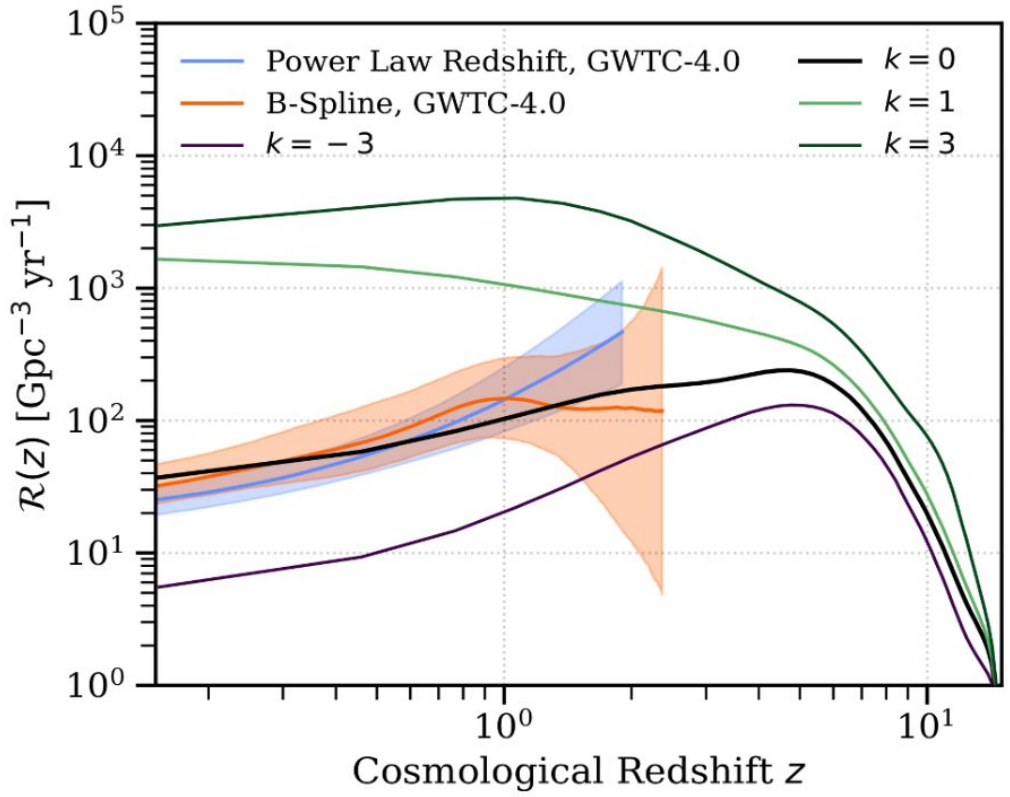


Un-normalized distribution of the BH masses merging within one year. The effect of k on the merger-mass distribution is evident, as different values of k **shift** the typical merger masses and increase the **number of events**.

PRELIMINARY

Constraints from GWTC-4 data

Total BBH merger rate density (MRD), Gpc^{-3} yr^{-1}



- **k = 0** peaks near z ≈ 5: star formation × metallicity × delay times
- **High z:** curves almost converge, mergers follow formation quickly
- **k > 0:** up to ~100× more mergers at z ≈ 0; the peak disappears
- **k < 0:** fewer low-z mergers, the drop steepens
- **Orbit term matters:** dropping −3kH(t) changes the k = 3 local rate by >10×

PRELIMINARY

Constraints from star clusters

ONGOING WORK led by I. Usai (UniCa), K. Kritos (Stony Brook, US)

We log-sample from a wide range in order to explore reasonable **globular cluster initial condition**.
We sample 145 clusters and we let them evolve according to 7 different values of k

METALLICITY
Z
Cluster metal content
1×10^{-4} – 1×10^{-2}

INITIAL MASS
M_{cl}
Total cluster mass
1×10^{4} – 3×10^{7} $M_\odot$

HALF-MASS RADIUS
r_h
Structural spatial extent
0.1 – 10 pc

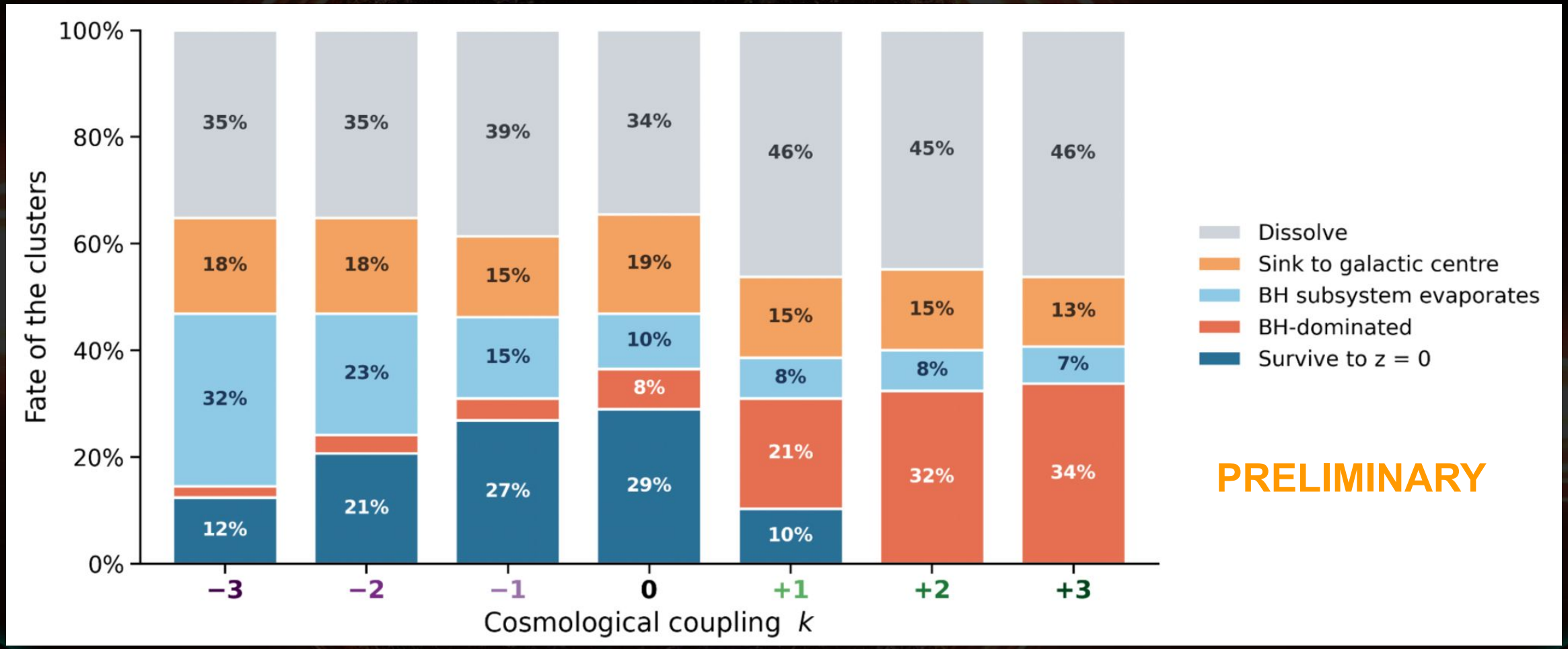
R
Distance to galactic center
1 – 10^5 pc

FORMATION REDSHIFT
Cosmic epoch of formation
$z \approx 2.4 – 13$

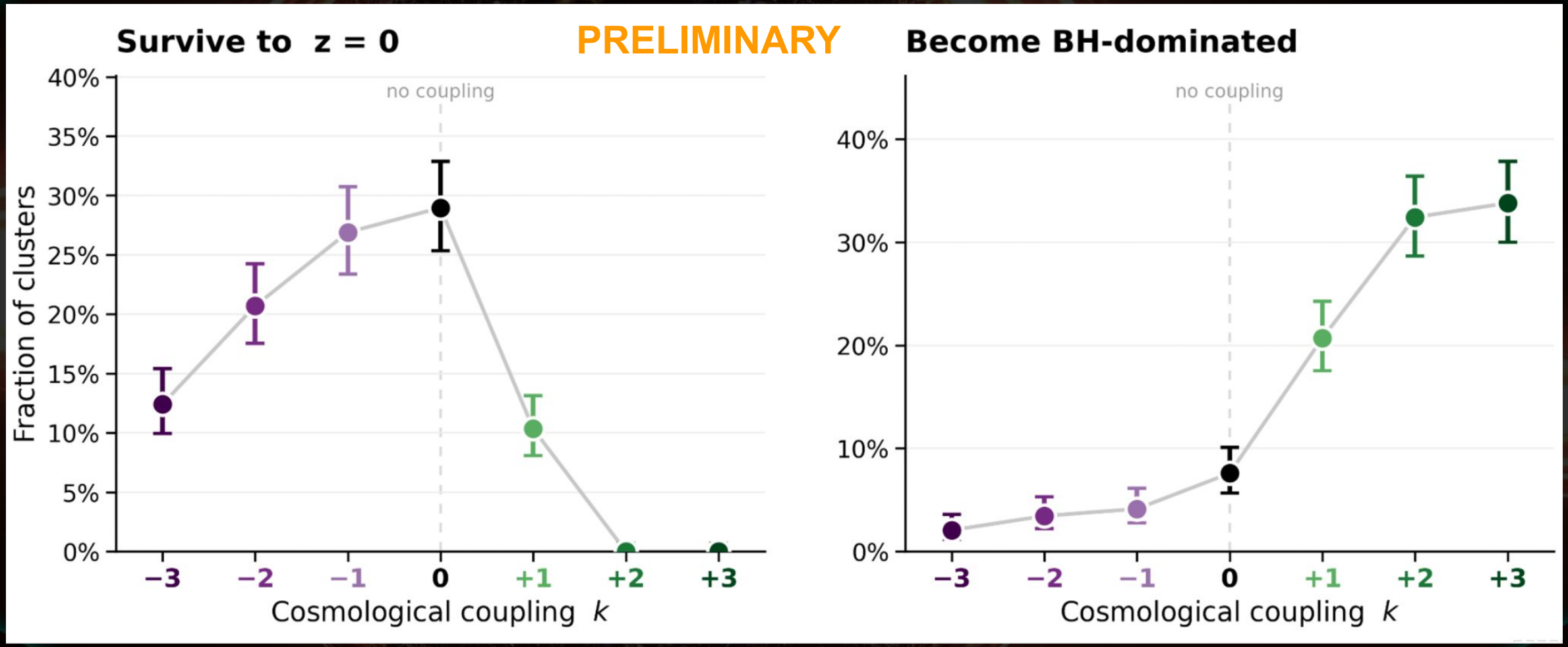


TESTED Latin-hypercube grids: 145 clusters × 7 values of k

Constraints from star clusters



Constraints from star clusters



Conclusions

- We demonstrate that the Misner-Sharp mass is the correct quantity at play in astrophysics and cosmology;
- We find that cosmological coupling is quite natural for compact objects sourced by local anisotropies like nonsingular black holes;
- We predict a universal linear growth of the MS mass as a function of redshift (with an additional subleading term for horizonless objects)

$$M(z) = M(z_i)\left(\frac{1+z_i}{1+z}\right)^k$$ with k = 1;

- We are now looking into data to figure out whether:
  - k = 0 ⟶ nonsingular GR BHs incompatible with observations
  - k = 1 ⟶ smoking gun that GR BHs are nonsingular
  - k = other value ⟶ astrophysical BHs cannot be described with GR

Is the large-scale dynamics influenced by late-time inhomogeneities?

Inhomogeneous anisotropic fluid cosmology and emergent dark energy

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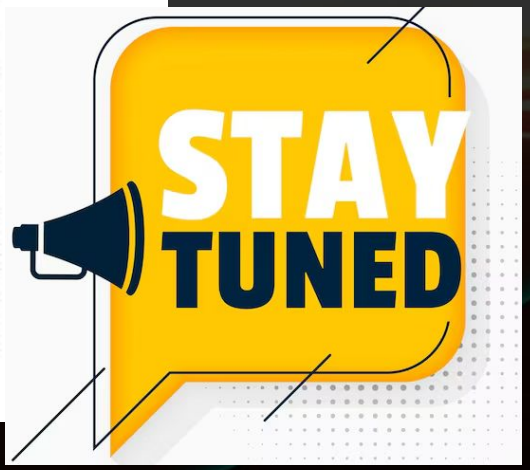
We propose a new framework for inhomogeneous cosmology based on General Relativity sourced by an anisotropic fluid. The model is parametrized by a single input function, the radial mass distribution at different redshifts, and admits exact solutions of Einstein's equations, avoiding the drawbacks of averaging, backreaction and fitting approaches. The Friedman-Lemaître-Robertson-Walker (FLRW) universe is recovered at large scales. The model provides a natural mechanism for the emergence of a cosmological constant Λ in the FLRW limit. The extensive term giving rise to Λ is not present in the exact metric solution, hence does not contribute to the local gravitational potential. It naturally emerges as a large-scale effect induced by the late-time inhomogeneous distribution of (baryonic and dark) matter. A striking feature of this mechanism is that it might provide a physical explanation for DESI's preference for an emergent dark energy component. We also discuss a simple, analytically tractable solution, and show that it reproduces the observed order of magnitude of Λ.



Thanks for your attention

Saludi e trigu a totus