Workshop for the 10th Anniversary of the Discovery of Gravitational Waves (28. Aug. 2025, SNU)

Testing New Rotating Black Hole Solutions in a Viable Lorentz-Violating Gravity: Observational Perspectives

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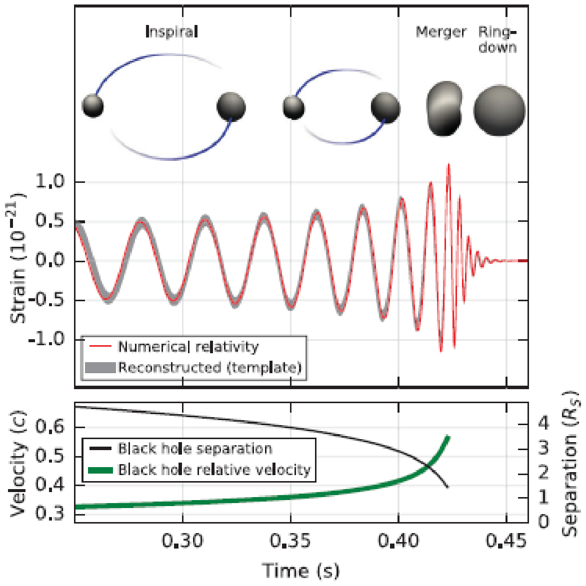
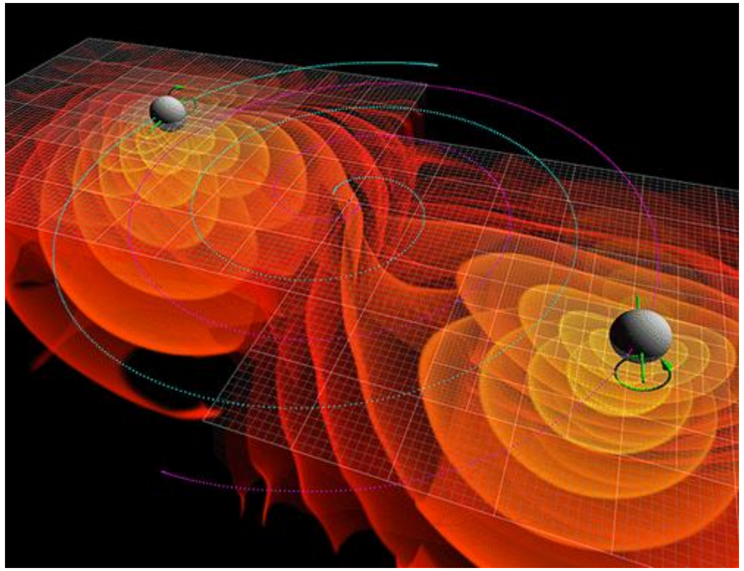
In coll. w/ Deniz O. Devecioglu, Hideki Maeda, ...

- 1. Rotating BH is believed to be real!



LIGO '15



- 2. The final state of collapse/collision is described Kerr solution (no-hair/uniqueness theorem)!!
- This is the importance of finding exact solutions in gravity theories.

- 3. But there are several evidences that GR is not enough for a (UV) complete description of our universe: Non-renormalizability, dark energy (and dark matter ?), Hubble tension (?), etc.

- 4. If there are Kerr-like rotating black hole solutions in another viable gravity model, we can use those as a laboratory for testing the gravity model, in comparison with Kerr in GR.

- 5. But it is extremely hard to get an “exact” rotating solution other than Kerr in GR (or its relativistic cousins), for example, in a UV-complete gravity with Lorentz-violation (Horava gravity).

- The only known solutions (D=4) are
- (1) D=4 non-rotating solutions (Lu-Mei-Pope, Kehagias-Sfetsos, Park (2009), Kiritsis-Kofinas (2010))
- (2) D=4 slowly rotating solution (Lee-Kim-Myung, Aliev-Senturk (2010))
- (3) D=4 numerical rotating solution for Einstein-Aether gravity [Adam et al., CQG 39, 125001 (2022)].

- 6. Recently, we have proposed a general procedure/strategy for finding exact rotating solutions but without (or less) tears!
- And, following the procedure, we found the exact (Kerr-like) rotating black hole solutions in low-energy Horava gravity.
- It took 15 yrs (2024. Feb) to get the solution after Horava gravity paper (2019, Jan.). We expect more years to get the rotating sol. for full Horava gravity.
- Cf. GR (1915); Schwarzshild (1915); Kerr (1963): 48 yrs

1. LOW-ENERGY HORAVA GRAVITY AND ROTATING BLACK HOLE SOLUTIONS

- The low energy (non-projectable) Horava gravity

$$S_g = \int_{\mathbf{R}\times\Sigma_t} dt d^3x \sqrt{g} N \left[\frac{1}{\kappa} \left(K_{ij}K^{ij} - \lambda K^2 \right) + \xi R - 2\Lambda + \frac{\sigma}{2} a_i a^i \right]$$

$$K_{ij} = (2N)^{-1} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right)$$

$$a_i = \nabla_i lnN$$

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

- 3 LV parameters $\lambda, \xi,$ and σ

- **EQN for** N, N^i, and g^{ij}

$$\mathcal{H} \equiv \frac{1}{\kappa}\left(K_{ij}K^{ij} - \lambda K^2\right) - \xi R + 2\Lambda - \sigma\left(\frac{1}{2}\frac{\nabla_i N \nabla^i N}{N^2} - \frac{\nabla_k \nabla^k N}{N}\right) = 0 ,$$

$$\mathcal{H}^i \equiv \frac{2}{\kappa}\nabla_j\left(K^{ji} - \lambda\, K g^{ji}\right) = 0 ,$$

$$E_{ij} \equiv \frac{1}{\kappa}\left(E_{ij}^{(1)} - \lambda E_{ij}^{(2)}\right) + \xi E_{ij}^{(3)} + \frac{\sigma}{2}E_{ij}^{(4)} = 0,$$

$$\begin{aligned} E_{ij}^{(1)} &= N_i \nabla_k K^k{}_j + N_j \nabla_k K^k{}_i - K^k{}_i \nabla_j N_k - K^k{}_j \nabla_i N_k - N^k \nabla_k K_{ij} \\ &\quad -2NK_{ik}K_j{}^k - \frac{1}{2}NK^{k\ell}K_{k\ell}\, g_{ij} + NKK_{ij} + \dot{K}_{ij} , \\ E_{ij}^{(2)} &= \frac{1}{2}NK^2 g_{ij} + N_i \partial_j K + N_j \partial_i K - N^k(\partial_k K) g_{ij} + \dot{K}\, g_{ij} , \\ E_{ij}^{(3)} &= N\Big(R_{ij} - \frac{1}{2}R g_{ij} + \frac{\Lambda}{\xi} g_{ij}\Big) - (\nabla_i \nabla_j - g_{ij}\nabla_k \nabla^k)N , \\ E_{ij}^{(4)} &= \frac{1}{N}\Big(-\frac{1}{2}g_{ij}\nabla_k N \nabla^k N + \nabla_i N \nabla_j N\Big) . \end{aligned}$$

- **With arbitrary $\lambda, \xi,$ and σ , the manifest symmetry of action is the foliation-preserving diffeomorphism**

$(Diff_{\mathcal{F}})$

$$
\begin{aligned}
\delta_\xi t &= -\xi^0(t), \quad \delta_\xi x^i = -\xi^i(t, \mathbf{x}), \\
\delta_\xi N &= (N\xi^0)_{,0} + \xi^k \nabla_k N, \\
\delta_\xi N_i &= \xi^0{}_{,0} N_i + \xi^j{}_{,0} g_{ij} + \nabla_i \xi^j N_j + N_{i,0}\xi^0 + \nabla_j N_i \xi^j, \\
\delta_\xi g_{ij} &= \nabla_i \xi^k g_{kj} + \nabla_j \xi^k g_{ki} + g_{ij,0}\xi^0.
\end{aligned}
$$

Here, the physical (gauge-invariant) quantities are K , $K_{ij}K^{ij}$, R , $R_{ij}R^{ij}$, $K_{ij}R^{ij}$, etc, capturing physical singularities!

- **Our ansatz with 3 undetermined functions,**

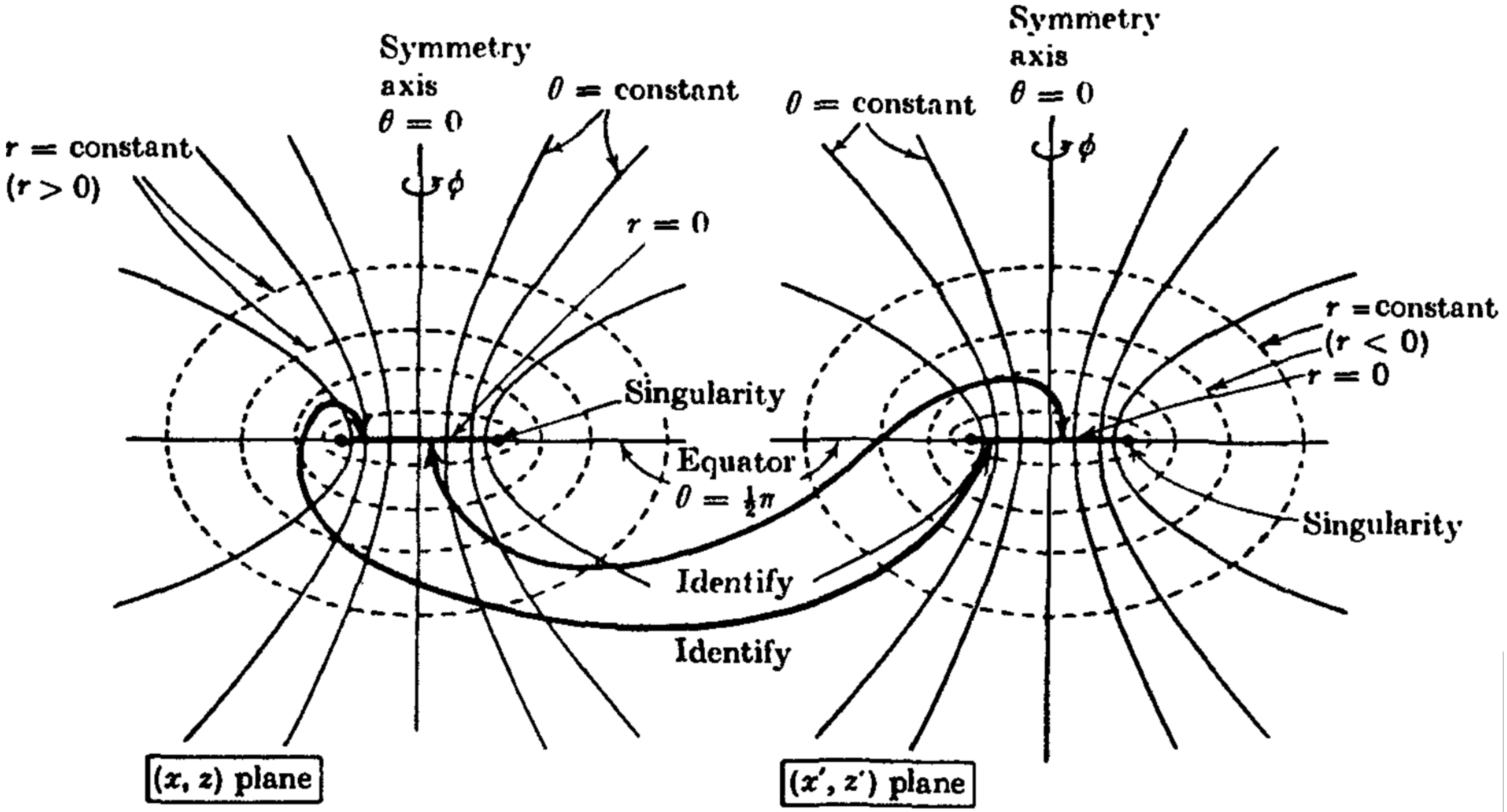
$f(r), g(r)$, and $\Delta_r(r)$ **:**

$$ds_1^2 = -N^2dt^2 + \frac{\rho^2}{\Delta_r}dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2 \sin^2\theta}{\rho^2}\left(d\phi + N^\phi dt\right)^2$$

$$\begin{aligned} \Sigma^2 &= \left(r^2 + a^2\right)\rho^2 + f(r)a^2\sin^2\theta, \\ N^2 &= \frac{\rho^2 \Delta_r(r)}{\Sigma^2}, \quad N^\phi = -\frac{g(r)}{\Sigma^2} \end{aligned}$$

Solution (with the usual choice $N^\phi|_\infty = 0, W(\infty) \equiv N\sqrt{g_{rr}}|_\infty = 1$):

$$f(r) = 2mr, \ \ g(r) = 2amr\sqrt{\kappa\xi}, \ \Delta_r(r) = r^2 + a^2 - 2mr,$$



Symmetry
axis
$\theta = 0$
ϕ
θ = constant
r = constant
(r > 0)
r = 0
Singularity
Equator
$\theta = \frac{1}{2}\pi$
Identify
Identify
(x, z) plane
Symmetry
axis
$\theta = 0$
ϕ
θ = constant
r = constant
(r < 0)
r = 0
Singularity
(x′, z′) plane

- Remarks:
- 1. It is rather surprising that the Kerr-solution cracking term with the LV factor κξ appears only in N^ϕ .
- 2. But, if we look at the component form, one can easily see the non-trivial LV effect for $\xi \neq 1/\kappa$ in g_{tt} as well as in $g_{t\phi}$ components!

$$ds_1^2 = \left[-\frac{(\Delta_r - a^2\sin^2\theta)}{\rho^2} + \frac{(\kappa\xi - 1)\,(2mr)^2\; a^2\sin^2\theta}{\rho^2\Sigma^2}\right] dt^2 + \frac{\rho^2}{\Delta_r}dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2\sin^2\theta}{\rho^2}d\phi^2 - \frac{4amr\sqrt{\kappa\xi}\,\sin^2\theta}{\rho^2}dtd\phi$$

- 3. Our solution, as well as the Kerr solution with $\xi = 1/\kappa$, are valid for an arbitrary λ due to $K = 0$, i.e., "maximal" slicing.

- This makes even Kerr solution (or Schwarzschild solution with a=0) has different notions of singularities, due to the lack of the full Diff with $\lambda \neq 1$.

2. SINGULARITY STRUCTURE

- **Curvature invariants** ($K = 0,\ K_{ij}R^{ij} = 0$)

$$R \sim \frac{a^2 m^2}{\rho^6 \Sigma^4},\quad R_{ij}R^{ij} \sim \frac{m^2}{\rho^{12}\Sigma^8},\quad K_{ij}K^{ij} \sim \frac{\kappa\xi a^2 m^2}{\rho^6 \Sigma^4}$$

show the curvature singularities at $\Sigma^2 = 0$, **as well as** $\rho^2 = 0$ (the usual ring singularity at r = 0, θ = π/2).

cf: 4D curvatures

$$R^{(4)} \sim (\kappa\xi - 1)\frac{a^2 m^2}{\rho^6 \Sigma^4},\quad R^{(4)}_{\mu\nu}R^{(4)\mu\nu} \sim (\kappa\xi - 1)^2 \frac{a^4 m^4}{\rho^{12}\Sigma^8},$$

$$R^{(4)}_{\mu\nu\sigma\rho}R^{(4)\mu\nu\sigma\rho} \sim (\kappa\xi - 1)\frac{m^2}{\rho^{12}\Sigma^8} + \frac{m^2}{\rho^{12}}(\cdots),$$

$$\Sigma^2 = \left(r^2 + a^2\right)\rho^2 + f(r)a^2\sin^2\theta$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$f(r) = 2mr$$

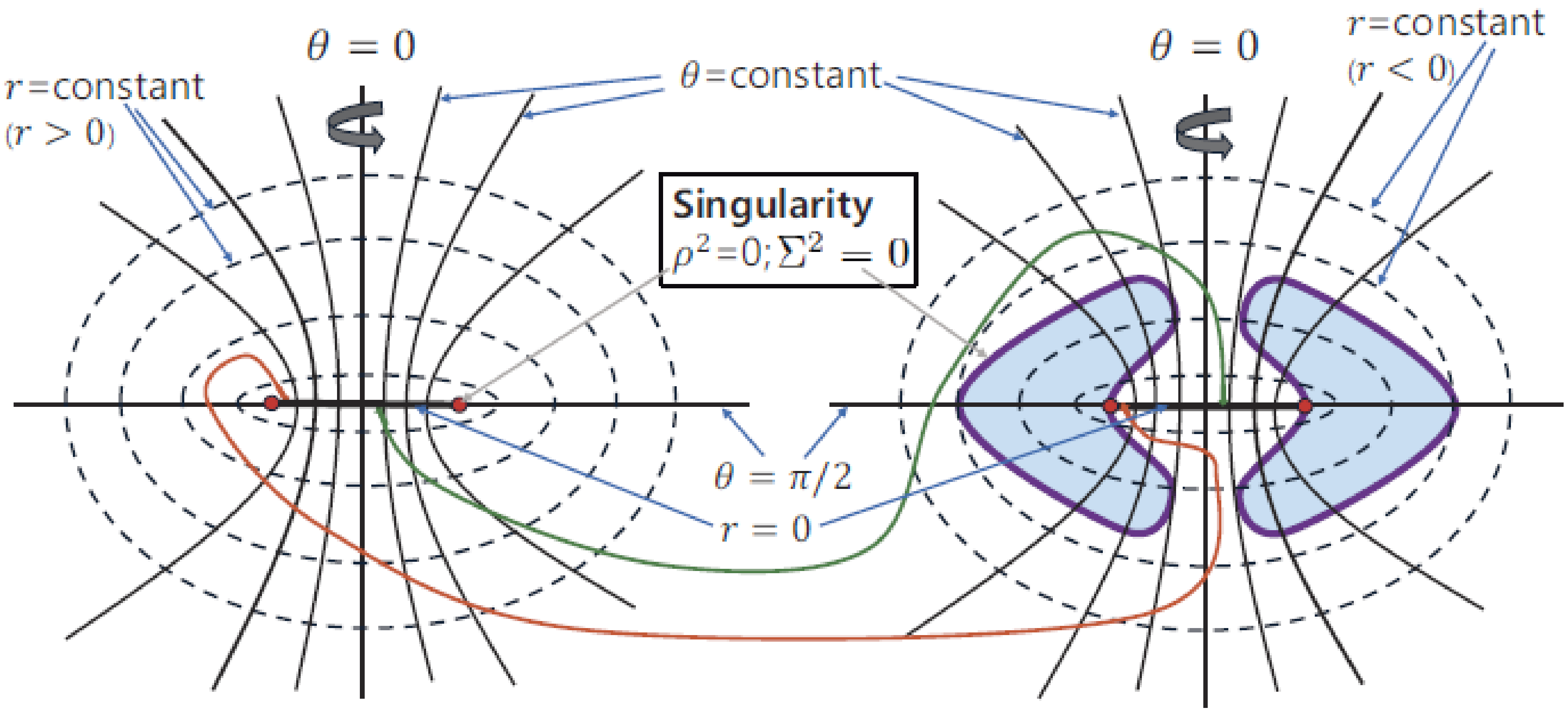


FIG. 2: The maximal extension of the rotating black hole solution for $m^2 > a^2$ by identifying the top of the disk $x^2 + y^2 < a^2, z = 0$ in $r > 0$ region (the left chart) with the bottom of the corresponding disk in $r < 0$ region (the right chart) and vice versa. The torus singularity exists at $\Sigma^2 = 0$ in $r \leq 0$ region that includes the usual ring singularity at $\rho^2 = 0$, *i.e.*, $r = 0, \theta = \pi/2$.

3. Other Properties.

- 1. There are two Killing horizons at

$$\Delta_r = r^2 + a^2 - 2ma = 0$$

as in GR!: The zeroth-law is satisfied

$$\chi^{[\mu}\nabla^{\nu]}_{(4)}\kappa_{\pm} = -\chi^{[\mu}R^{\nu]}{}_{\sigma(4)}\chi^{\sigma} = -\chi^{[\mu}T^{\nu]}{}_{\sigma(eff)}\chi^{\sigma} = 0,$$

$$T^{\mu}{}_{\nu(eff)} = \begin{pmatrix} \widehat{\rho} & 0 & 0 & 0 \\ 0 & \widehat{p_1} & \widehat{p_2} & 0 \\ 0 & \widehat{p_3} & -\widehat{p_1} & 0 \\ \widehat{p_4} & 0 & 0 & -3\widehat{\rho}, \end{pmatrix}$$

- **2. Hamilton-Jacobi equation is not separable!**

$$
\begin{aligned}
-2\frac{\partial S}{\partial \tau} &= g^{(4)\mu\nu}\partial_\mu S \partial_\nu S \\
&= -\frac{\Sigma^2}{\rho^2 \Delta_r}(\partial_t S)^2 + \frac{4mra\sqrt{\kappa\xi}}{\rho^2\Delta_r}\partial_t S \partial_\phi S + \frac{\Delta_r \rho^4 - (2mra)^2 \kappa\xi \sin^2\theta}{\Sigma^2 \rho^2 \Delta_r \sin^2\theta}(\partial_\phi S)^2 \\
&\quad + \frac{\Delta_r}{\rho^2}(\partial_r S)^2 + \frac{1}{\rho^2}(\partial_\theta S)^2
\end{aligned}
$$

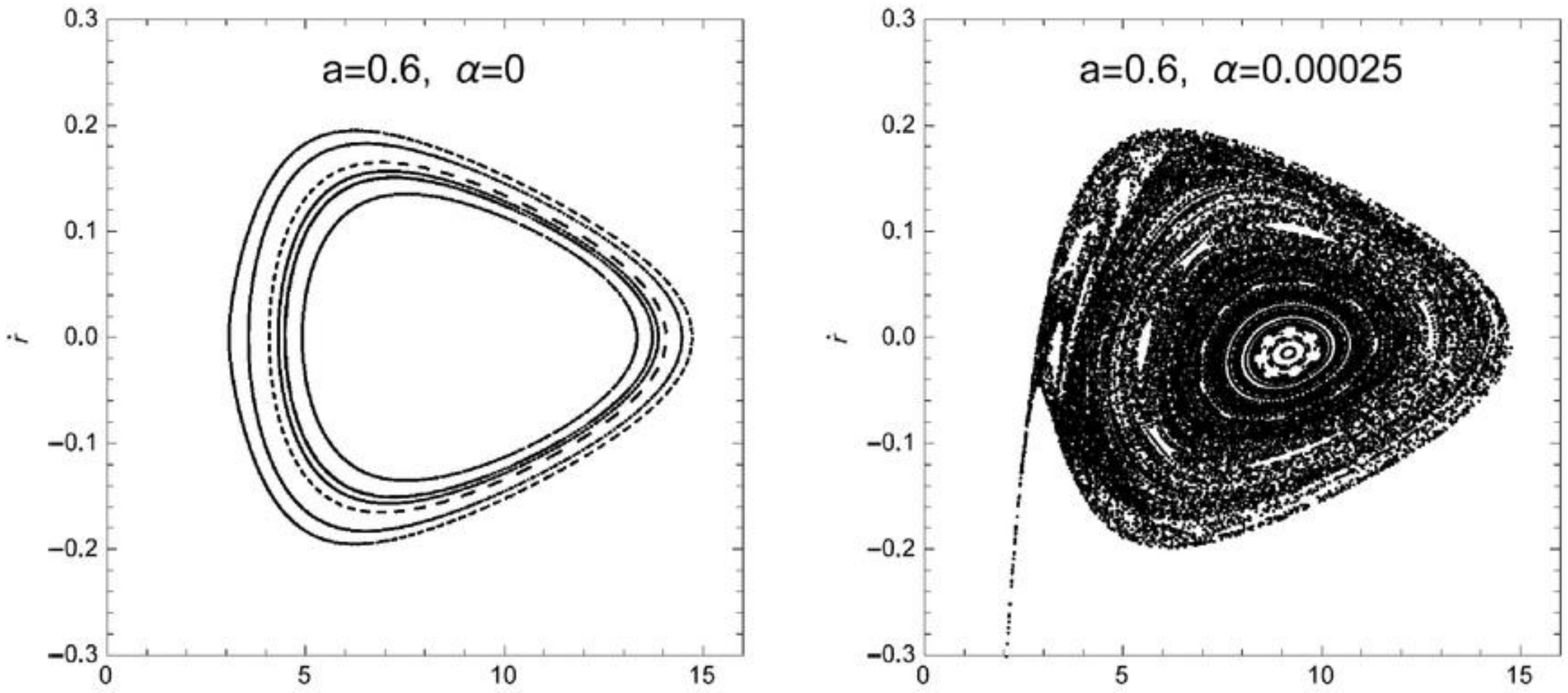
from a particle Hamiltonian $H = (1/2)g^{\mu\nu}p_\mu p_\nu$ and $p_\mu = \partial_\mu S$ with the Hamilton's principal function S. By "assuming" the separability, we consider [2]

$$
S = \frac{1}{2}\mu^2\tau - Et + L\phi + S_r(r) + S_\theta(\theta) \tag{D4}
$$

$$
\left\{\mu^2 r^2 + \left(aE + \sqrt{\kappa\xi}L\right)^2 - \frac{1}{\Delta_r}\left((r^2+a^2)E + \sqrt{\kappa\xi}aL\right)^2 + \Delta_r\left(\frac{dS_r}{dr}\right)^2 + (\kappa\xi - 1)\frac{a^2L^2}{\Delta_r}\right\}
$$
$$
+ \left\{\mu^2 a^2 \cos^2\theta + \left(-a^2E^2 + \frac{\kappa\xi L^2}{\sin^2\theta}\right) + \left(\frac{dS_\theta}{d\theta}\right)^2 - (\kappa\xi - 1)\frac{L^2}{\sin^2\theta}\right\} + (\kappa\xi - 1)\frac{(2mra)^2 L^2}{\Sigma^2\Delta_r} = 0,
$$

- **Implications:**

Non-separable, i.e., non-integrable, HJ might indicate Chaotic behaviors, i.e., Poincare section!



- 3. For non-separable HJ, we need purely numerical analysis (i.e., ray tracing method) to obtain (theoretical) shadows, contrary to Kerr, whose HJ is separable and so analytic formula can be found.

- In 2025 June [arXiv: 2506.13504v1 [gr-qc]], there was the first shadows analysis and observation test. It shows peculiar patterns of shadows depending on LV parameter.

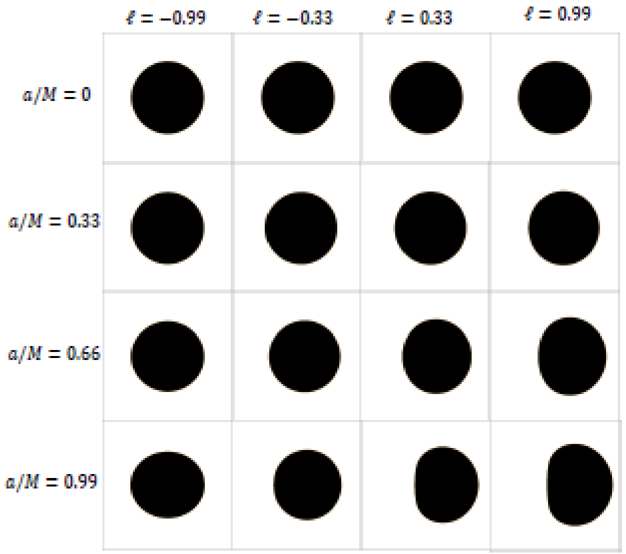


FIG. 1. Shadows cast by rotating Hořava BHs, as seen by an observer at $\theta_0 = \pi/2$.

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Letter

Lorentz violation signatures in the low-energy sector of Hořava gravity from black hole shadow observations

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ABSTRACT

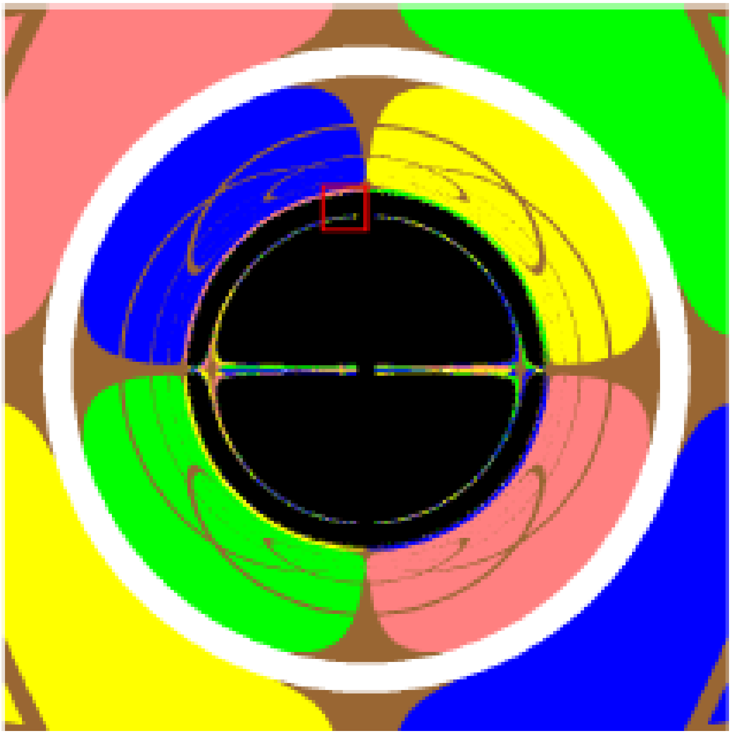
In this paper, we use the Hořava gravity model and EHT observations of supermassive black holes (BHs) to investigate signatures of Lorentz violation in real astrophysical environments. The Lorentz violation in the rotating Hořava BH spacetime are confined to the strong gravitational field region, being induced by the BH's rotation. Due to the non-separability of the photon motion equations in this spacetime, we employed a numerical backward ray-tracing method to generate shadow images for various BH parameters. Subsequently, we extracted coordinate positions characterizing the shadow shape from high-pixel images to evaluate the parameter space of the BH. When evaluating M87*, Lorentz violation can occur with arbitrary magnitude. However, for Sgr A*, we can impose certain parameter constraints on Lorentz violation. These constraints depend on the BH's spin. Should future observations at the highest inclination angle of the spin axis determine that the spin parameter of Sgr A* is less than 0.6, general relativity would be challenged, and the current EHT observations will serve as crucial evidence for the existence of Lorentz violation in low-energy environments.

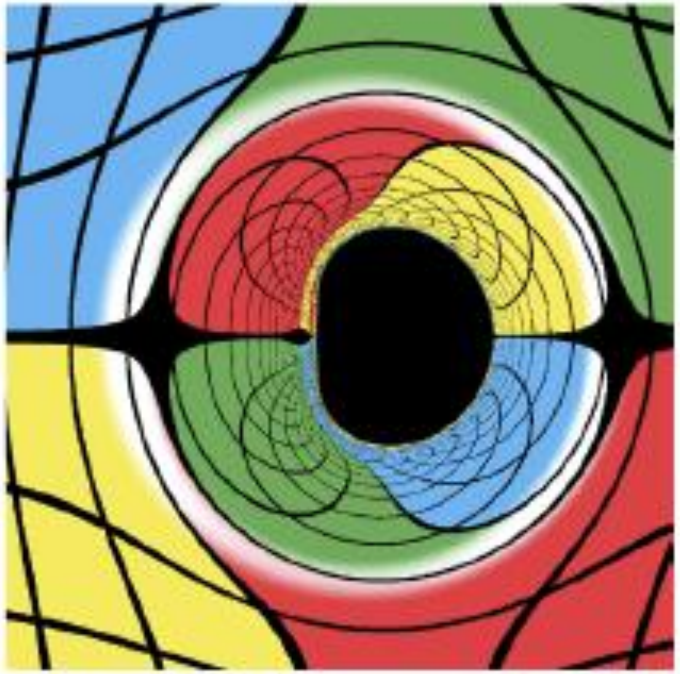
- The analaysis shows a slight preference of LV solution, though it might depend on accretion model for EHT observation and more extensive analysis is needed (needs high CPU times).

A B S T R A C T

In this paper, we use the Hořava gravity model and EHT observations of supermassive black holes (BHs) to investigate signatures of Lorentz violation in real astrophysical environments. The Lorentz violation in the rotating Hořava BH spacetime are confined to the strong gravitational field region, being induced by the BH's rotation. Due to the non-separability of the photon motion equations in this spacetime, we employed a numerical backward ray-tracing method to generate shadow images for various BH parameters. Subsequently, we extracted coordinate positions characterizing the shadow shape from high-pixel images to evaluate the parameter space of the BH. When evaluating M87*, Lorentz violation can occur with arbitrary magnitude. However, for Sgr A*, we can impose certain parameter constraints on Lorentz violation. These constraints depend on the BH's spin. Should future observations at the highest inclination angle of the spin axis determine that the spin parameter of Sgr A* is less than 0.6, general relativity would be challenged, and the current EHT observations will serve as crucial evidence for the existence of Lorentz violation in low-energy environments.

- **With higher precision of shadow analysis (theory, observation), chaotic behaviors could be seen also.**





an observer at $\theta_0 = \pi/2$.

- **(cf. no chaotic signature in the recent shadow analysis by Liu et al. (2025), due to precision limit)**

- For beyond GR test in EHT, dilaton-haired (rotating) solution (Sen's solution), which is low-energy limit of string theory has been considered for machine learning.

- For a viable quantum gravity, we need to consider our rotating solution, which is a simple vacuum solution without hairs. But for that, we first need to teach our solution to AI !

Deep learning inference with the Event Horizon Telescope

I. Calibration improvements and a comprehensive synthetic data library

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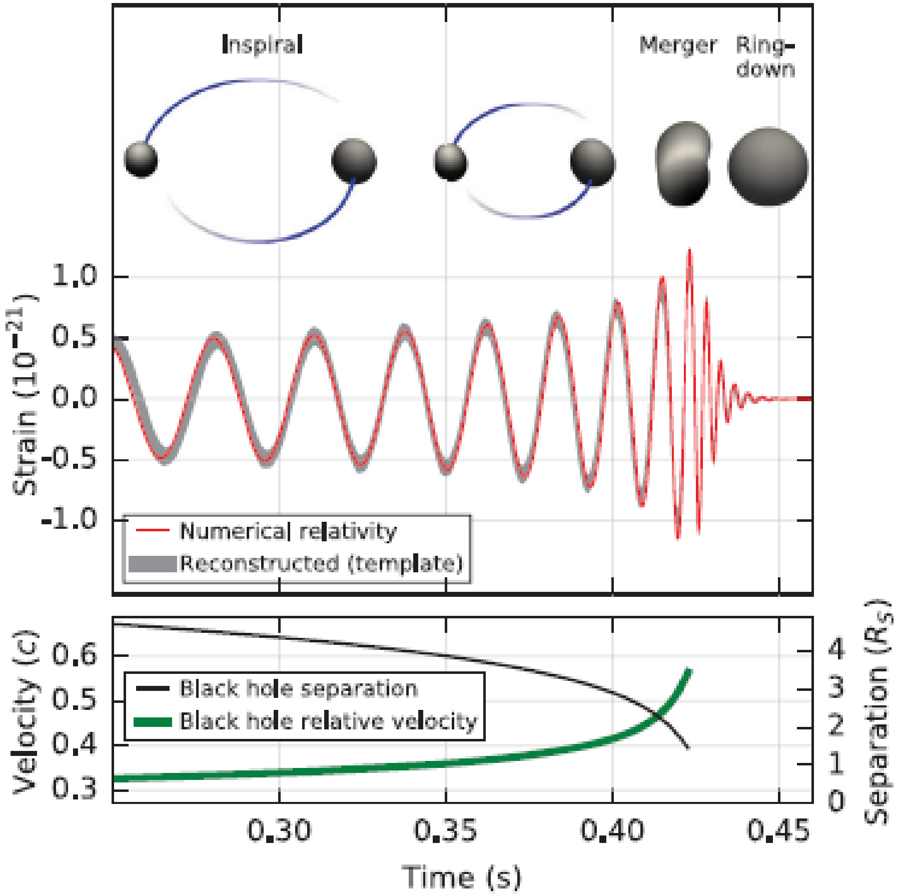
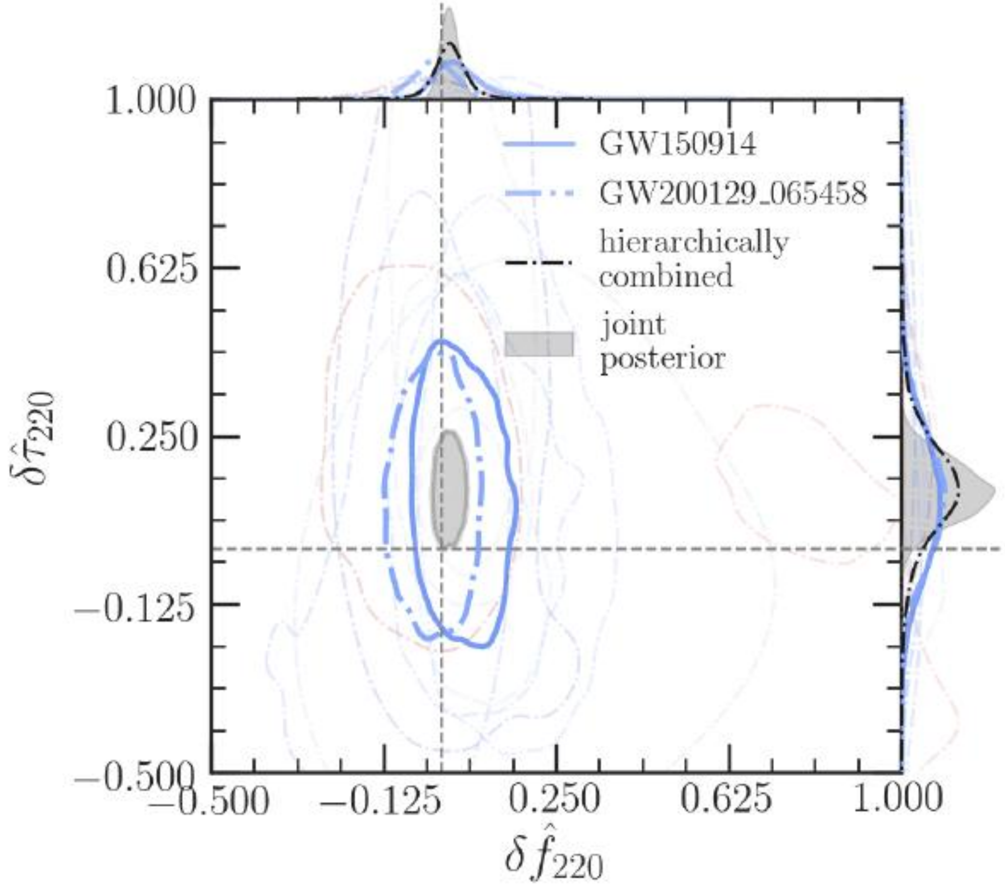
4. GW tests ?

- **Quasi-Normal Modes** (QNMs) test (up to **O3**):





- **To test QNMs for our rotating solution, we first need to QNMs by**
- **(1) obtaining Teukolsky-like equation for black hole perturbations, and then**
- **(2) solving it numerically.**

- **For charged black holes, rotating balck holes exist only for a novel coupling $\zeta\eta^{-1} = \kappa\xi.$ with the LV Maxwell action,**

$$S_M = \int_{\mathbf{R}\times\Sigma_t} dtd^3x\sqrt{g}N\left[-\frac{2\eta}{N^2}\left(E_i + F_{ij}N^j\right)^2 + \zeta F_{ij}F^{ij}\right]$$

- **This produces the same speed of graviton and light (em) $c_g = c_l = \sqrt{\kappa\xi}$**
- **In this case, the arrival delay of (+1.74 ± 0.05)s in the coincident gravitational waves (GW) and gamma rays (GW170817, GRB170817A)**

$$-3\times 10^{-15} c_l < (c_g - c_l) < 7\times 10^{-16} c_l,$$

- **does not mean $-3\times 10^{-15} < \Delta c_g/c < 7\times 10^{-16}$ with $\Delta c_l = 0$ (Here, $c_g = c + \Delta c_g,\ c_l = c + \Delta c_l.$)**

- **Message:**
- **1. The different speed of graviton and light (em) is not allowed in (low-energy limit of) Horava gravity. This might imply the same universal speed limit for gravity and EM parts, due to some consistent interactions between them, though not constrained in the action.**

- **2. The will be due to different different speed of graviton and light might be due to different origins, like as, different interaction with environment, different generation time, etc…**
- **(cf. Zong-Hong Zhu's talk)**

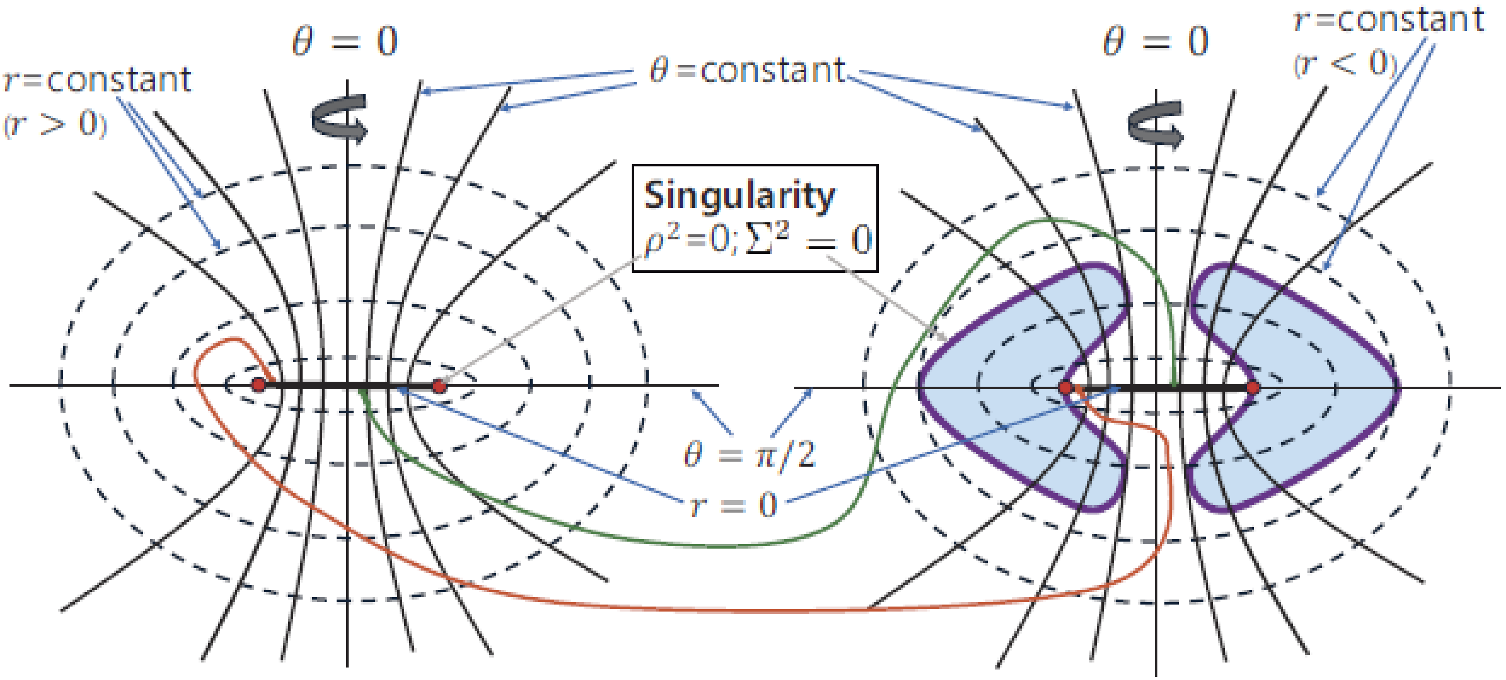
5. Desirable Inner Structure [coll. w/ H. Maeda]: No Closed Time-like Curves (CTC)!

- In Kerr solution, there is causality-violating region (closed time-like curves (CTC)).

$$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right) dt^2 - \frac{2GMar\sin^2\theta}{\rho^2}(dt\, d\phi + d\phi\, dt) + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2 + a^2)^2 - a^2\Delta\sin^2\theta\right] d\phi^2,$$

$$ds_1^2 = \left[-\frac{(\Delta_r - a^2\sin^2\theta)}{\rho^2} + \frac{(\kappa\xi - 1)\ (2mr)^2\ a^2\sin^2\theta}{\rho^2\Sigma^2}\right] dt^2 + \frac{\rho^2}{\Delta_r}dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2\sin^2\theta}{\rho^2}d\phi^2 - \frac{4amr\sqrt{\kappa\xi}\ \sin^2\theta}{\rho^2}dtd\phi$$

- CTC region is protected by the new singularity at $\Sigma^2 = 0$, : Just the mere existence of LV coupling can protect CTC region!!; Chronology Protection!!
- In Kerr, there is the same CTC region but one can freely move across $\Sigma^2 = 0$,, which is not singularity in GR !



5. Summary

- 1. We found a new non-Kerr exact vacuum solution (2024), which has a more desirable inner structure (no CTC problem).
- 2. It will be an important role in the Beyond-GR test in the near future.
- 3. We need to prepare the future observations with higher precision, (1) GW templates, (2) numerical relativity code, (3) Machine learning, (4) QNMs, etc, for our new rotating solution.